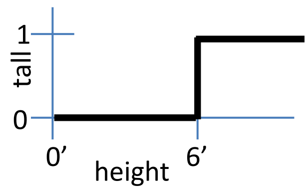
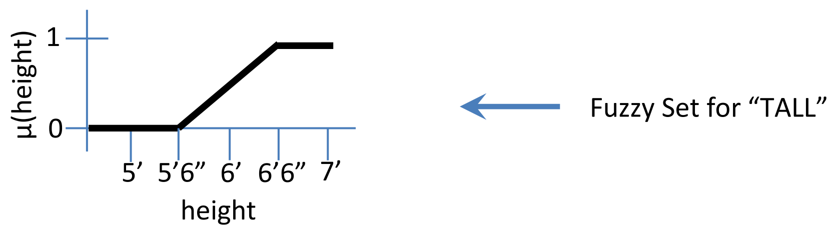
Fuzzy Logic - Basics

Standard Boolean logic:



Fuzzy logic:



"μ" is the "<u>degree of membership</u>" of the <u>variable</u> "height" in the <u>fuzzy set</u> "TALL".

"<u>Crisp</u>" values for "height" are measured (e.g.: 5'6"). The corresponding μ is its <u>fuzzy</u> membership.

"<u>Linguistic variables</u>": In the above example, "height" is a linguistic variable. Another example: "speed"
"<u>Linguistic values</u>": In the above example, "tall" is a linguistic value. Other examples: "fast", "loud", etc.

Fuzzy logic operations:

```
NOT(x)    ≡  1 - μ(x)
x AND y   ≡  min(x,y)
x OR y    ≡  max(x,y)
```

Mamdani-style Fuzzy Expert System:

1. **"Fuzzification"**
   determine the degree of membership for each input in the antecedent fuzzy sets.

2. **"Rule Evaluation"**
   combine antecedents using fuzzy logic operations (AND, OR, NOT)

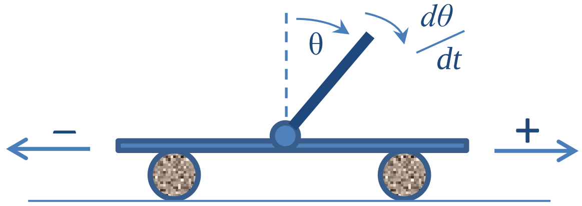
3. **"Aggregation"**
   express consequents as a single (aggregate) fuzzy set

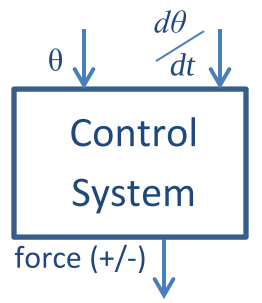
4. **"Defuzzification"**
   determine weighted sum, usually using "centroid" method. Reimann sum often useful.

$$\frac{\int \mu(x)\cdot x}{\int \mu(x)} \qquad \text{or, estimated with:} \qquad \frac{\sum \mu(x)\cdot x}{\sum \mu(x)}$$

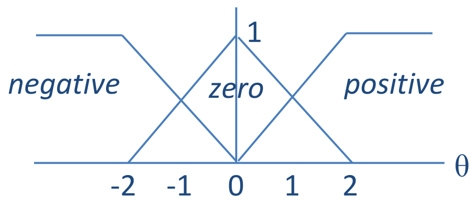
Fuzzy Expert System *(Mamdani-style example)*

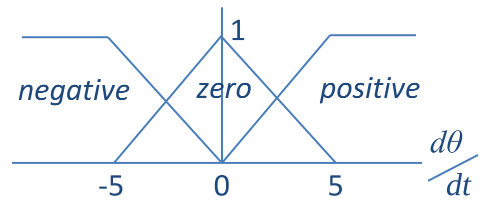
Application: *<u>Pole-Balancing control system</u>* *(Luger, 2005)*





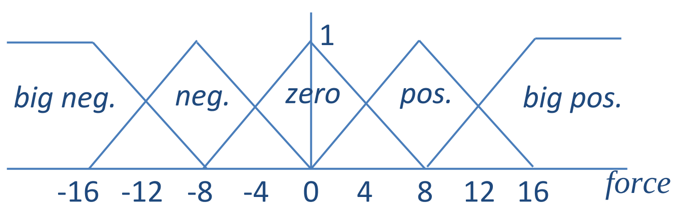
Let's make the <u>inputs</u> fuzzy:





i.e., for θ, negative=(-2/1,0/0), zero=(-2/0,0/1,2/0), positive=(0/0,2/1)
for dθ, negative=(-5/1,0/0), zero=(-5/0,0/1,5/0), positive=(0/0,5/1)

Let's also make the <u>output</u> fuzzy:



i.e., for force, bigneg=(-16/1,-8/0), neg=(-16/0,-8/1,0/0), zero=(0/0,8/1,16/0),
pos=(0/0,8/1,16/0), bigpos=(8/0,16/0).

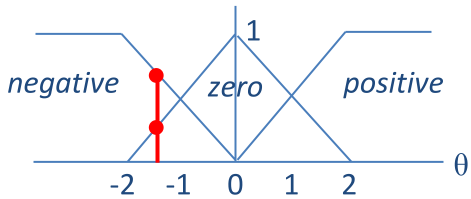
Rules can be expressed as a Fuzzy Associative Matrix (FAM):

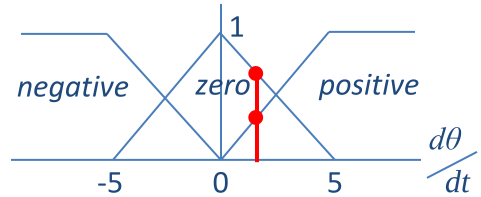
θ \ $d\theta/dt$	POS	ZERO	NEG
POS	BP	P	Z
ZERO	P	Z	N
NEG	Z	N	BN

(9 rules)

Now, let's consider a particular scenario, with θ = -1.5, and dθ = 2.

STEP 1: Fuzzification





$\mu_{pos}(\theta) = 0$

$\mu_{zero}(\theta) = 0.25$

$\mu_{neg}(\theta) = 0.75$

$\mu_{pos}(d\theta) = 0.4$

$\mu_{zero}(d\theta) = 0.6$

$\mu_{neg}(d\theta) = 0$

STEP 2: Rule Evaluation

We evaluate every rule in the FAM:

IF θ = pos (0) AND dθ = pos (.4) → force = BP (0)	IF θ = pos (0) AND dθ = zero (.6) → force = P (0)	IF θ = pos (0) AND dθ = neg (0) → force = Z (0)
IF θ = zero (.25) AND dθ = pos (.4) → force = P (.25)	IF θ = zero (.25) AND dθ = zero (.6) → force = Z (.25)	IF θ = zero (.25) AND dθ = neg (0) → force = N (0)
IF θ = neg (.75) AND dθ = pos (.4) → force = Z (.4)	IF θ = neg (.75) AND dθ = zero (.6) → force = N (.6)	IF θ = neg (.75) AND dθ = neg (0) → force = BN (0)

Note that rule evaluation is done using the fuzzy definition of AND:

$$X \textbf{ AND } Y = \textbf{MIN}(\mu(X), \mu(Y))$$

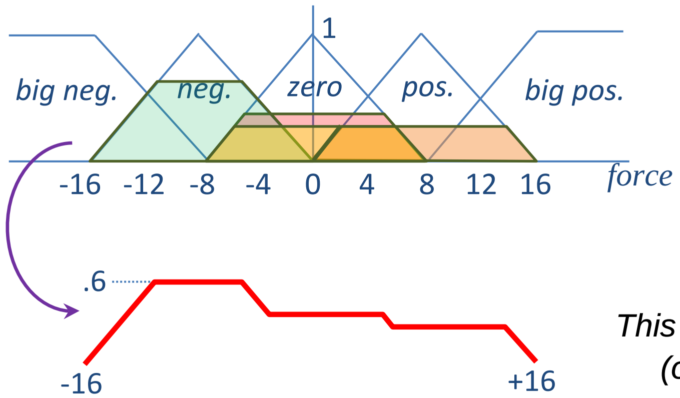
The circled rules have a non-zero output membership - they "fire".

But they are contradictory:

```
force = P (.25)
force = Z (.25)   ???
force = Z (.4)    ???
force = N (.6)
```

How do we resolve this?

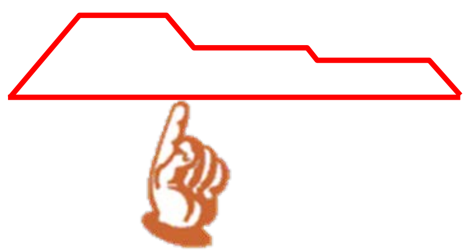
STEP 3: <u>Aggregation</u>



This is a <u>fuzzy</u> result (obviously!)

STEP 4: <u>Defuzzification</u>

Find the *centroid* of the above figure.
Or, at what point along the x-axis would such an object "balance"?



This can be calculated exactly using integration by parts (not easy!).
Or it can be estimated satisfactorily using a Riemann Sum, such as:

$$\frac{(-16*0)+(-12*.5)+(-8*.6)+(-4*.6)+(0*.4)+(4*.4)+(8*.25)+(12*.25)+(16*0)}{0 + .5 + .6 + .6 + .4 + .4 + .25 + .25 + 0}$$

Which equals **`-2.2`** in this case. This is a <u>crisp</u> result.
The controller thus sends a small negative force of `-2.2` to the cart.

This is reasonable, since the inputs indicated that the pole was leaning heavily to the left, but was also moving to the right at a modest speed. The correction that is already underway probably isn't quite enough to straighten the pole completely, so a slight additional correction by moving the cart a bit to the left makes sense.