1. 25 pt's. A student has designed a 12V dc motor driven potter’s wheel as shown below. \( J_c \) is the inertia of the clay that is to be molded into a pot. The student calculated, using the motor data sheet, \( \frac{k_f}{R_a} = 8 \) and \( k_b = 0.02 \).

\[
\begin{align*}
J_c &= 2 \text{ kg} \cdot \text{m}^2 \\
N_m &= 200 \\
N_c &= 10 \\
J_a &= 0.01 \text{ kg} \cdot \text{m}^2 \\
D_a &= 0.2 \frac{\text{N} \cdot \text{m} \cdot \text{s}}{\text{rad}}
\end{align*}
\]

Foot Pedal Controls the Motor Voltage Command

a) Find \( \frac{\theta_c(s)}{E_a(s)} \).

b) How much time does it take for the motor to reach approximately 2/3 of the final speed it reaches for a constant input voltage?

6) Move \( J_c \) to motor shaft \( \Rightarrow J_{cm} = \left( \frac{N_m}{N_c} \right)^2 \frac{J_c}{J_a} = \left( \frac{200}{10} \right)^2 \frac{2}{800} = 800 \)

\[
\begin{align*}
J_m &= J_a + J_{cm} = 0.01 + 800 = 800.01 \\
D_m &= D_a = 0.2
\end{align*}
\]

\[
\frac{\theta_m(s)}{E_a(s)} = \frac{8/800.01}{s(s + \frac{1}{800.01}(0.2 + (8)(0.02)))} = \frac{8}{s(800.015 + 0.36)}
\]

\[
\frac{(\frac{1}{20})\theta_c(s)}{E_a(s)} = \frac{8}{s(800.015 + 0.36)} = \frac{160}{s(800.015 + 0.36)} = \frac{444.44}{s(2222.255 + 1)}
\]

\[
T_{sys} = 2222.25 \text{ sec}. \text{ IT TAKES 37 MINUTES FOR THE POTTER'S WHEEL TO SPIN UP TO 2/3 OF THE DESIRED VELOCITY.}
\]

by just switching the gears, we achieve \( T_{sys} = 0.31 \text{ sec.} \) OR, the motor is undersized for the task, speed of \( \theta_c \) needs to be matched to speed of motor.
2. 25 pt's Given the following state-space model of a system, find $\frac{Y(s)}{U(s)} = G_{CL}(s)$. Once you have found a common denominator, you do not need to simply it any further. Also, you do not need to put the denominator into polynomial form.

$$
\begin{align*}
\dot{x} &= \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} 3 & 0 \end{bmatrix} x + 3 u \\
\end{align*}
$$

\begin{align*}
G_{CL}(s) &= C (sI - A)^{-1} B + D \\
&s = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} \\
&(sI - A)^{-1} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 5+4 & 0 \\ 0 & 5+4 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} (5+2) & 0 \\ 0 & (5+4) \end{bmatrix} \\
&= \frac{(5+2)}{(5+4)(5+2)} \\
G_{CL}(s) &= \begin{bmatrix} 3 & 0 \end{bmatrix} \begin{bmatrix} \frac{(5+2)}{(5+4)(5+2)} & 0 \\ 0 & \frac{(5+4)}{(5+2)} \end{bmatrix} + 3 \\
&= \frac{(5+2)}{(5+4)} + 3 \\
&= \begin{bmatrix} 3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{5+2} \end{bmatrix} + 3 \\
&= \begin{bmatrix} 0 + 3 \end{bmatrix} \\
G_{CL}(s) &= 3
\end{align*}

3. 25 pt's. Given the following closed-loop system, \( G_{CL}(s) = \frac{\omega_n^2(s + 1000 + 3j)(s + 1000 - 3j)}{(s^2 + 2\delta \omega_n s + \omega_n^2)} \)

a) Assuming the system above can be approximated as a 2\(^{nd}\) order system, design \( G_{CL}(s) \) so that the output has a percent overshoot be 1% and the settling time, \( T_s \), of 20 ms. Fill in all of the unknown parameters in \( G_{CL}(s) \).

b) Is our assumption correct that the closed-loop system above can be approximated as a 2\(^{nd}\) order system? Explain your answer and describe the effect of the extra poles and/or zeros.

\[
\phi = \frac{-\ln(0.01)}{\sqrt{\pi^2 + \ln(0.01)^2}} = 0.826 \Rightarrow \text{UNDERDAMPED, COMPLEX POLES}
\]

\[
T_s = \frac{4}{\delta \omega_m} \Rightarrow \omega_m = \frac{4}{T_s} \cdot \frac{4}{0.02 \times 0.826} = 242.13 \text{ rad/ sec}
\]

\[
G_{CL}(s) = \frac{(242.13)^2(s + 10^2 + 3^2)(s + 10^2 - 3^2)}{s^2 + 400s + (242.13)^2}
\]

\[
\rho_{1,2} = -\delta \omega_m \pm j\delta \omega_m \sqrt{1 - \delta^2} = -200 \pm j136.5
\]

\[
\text{Real Zero Part} = -1000 \leq s(-200) = -1000 = \text{Therefore,}
\]

\( G_{CL}(s) \) can be approximated as a 2\(^{nd}\) Order System.

As a result, we assume the zeros do not effect the system response.
4. 25 pt's. a) Convert the following system block diagram to a Signal-Flow-Graph and b) find \( \frac{C(s)}{R(s)} = G_{CL}(s) \) using Mason's Rule.

1 Loop: 
\[ a = -G_1(G_2 + G_3)H_1H_2 \]

2 Forward Paths 
\[ T_1 = G_1(G_2 + G_3) \]
\[ T_2 = G_4H_2G_1(G_2 + G_3) \]

\[ \Delta = 1 - a \]
\[ a \text{ touches } T_1 \]
\[ \Delta_1 = 1 \]

\[ a \text{ touches } T_2 \]
\[ \Delta_2 = 1 \]

\[ \frac{C(s)}{R(s)} = G_{CL}(s) = \frac{T_1 + T_2}{\Delta} \]