where \( J_w = 0.020 \text{ kg} \cdot \text{m}^{-2} \), \( D_{w1} = 0.01 \text{ N-M/s/rad} \), \( D_{w2} = 0.02 \text{ N-M/s/rad} \), \( J_a = 0.01 \text{ kg} \cdot \text{m}^{-2} \),

\( D_a = 0.01 \text{ N-M/s/rad} \), \( N_m = 10 \), \( N_w = 100 \). The student also calculated, using the data sheet, \( \frac{k_T}{R_a} = 3 \) and \( k_b = 0.01 \).

a) Find \( \frac{\theta_m(t)}{E_a(t)} \).

b) What is the 1st order time constant of \( \frac{\theta_m(t)}{E_a(t)} = \frac{\theta_m(s)}{E_a(s)} \)?

\[
\frac{\theta_m(s)}{E_a(s)} = \frac{\theta_m(s)}{\theta_m(s)} = \frac{s}{s + \frac{1}{\tau_m}}
\]

\[
\tau_m = \frac{D_a}{J_m} = \frac{0.01}{0.01} = 1 \text{ s}
\]

\[
\theta_m(t) = \frac{\theta_m(0)}{e^{-t/\tau_m}}
\]

\[
\theta_m(t) = \frac{10E_a(t)}{e^{-t/\tau_m}}
\]

\[
\theta_m(t) = \frac{10E_a(t)}{e^{0.1t}}
\]

\[
\tau_m = \frac{3}{0.01} = 300 \text{ s}
\]

\[
\theta_m(t) = \frac{29.41}{\theta_m(t)} = \frac{7.44}{5(0.25t + 1)}
\]

\[
\theta_m(t) = \frac{7.44}{\theta_m(t)} = \frac{7.44}{0.25t + 1}
\]

\[
\tau_m = 0.25 \text{ seconds}
\]
2. 25 pt's. Given \[ \frac{C(s)}{R(s)} = G_{CL}(s) = \frac{4s^3 + 12s^2 + 12}{2s^3 + 6s^2 + 2} = 2 + \frac{8}{2s^3 + 6s^2 + 2} \]

find a Phase Variable state representation of the given system.

\[ G_{CL}(s) = 2 + \frac{4}{s^3 + 3s^2 + 0.5 + 1} \]

\[ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t) \]

\[ y = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 2 \cdot r(t) \]

Will accept a special case

\[ B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \]
a) Assuming the plot represents, approximately, the output response of a 2nd order system find: 1) the damping ratio, 2) the natural frequency, and 3) the poles of $\frac{C(s)}{R(s)} = G_C(s)$.

b) However, the actual $G_C(s)$ is $G_C(s) = \frac{j0^+ (5.354s + 280)}{(s + 200)(s^2 + 260s + 0.3)}$. Are we correct in assuming that the plot represents the response of a 2nd order system? Justify your Yes or No answer.

b) $G_C(s) = \frac{2}{s} \Rightarrow 0.05 = (3 - 2)/10 = 0.05$,

\[ s = -\ln(0.05)/\ln(1.2) \approx 8.7 \]

\[ \frac{5}{2} < 1 \Rightarrow \text{undamped} \]

\[ \frac{5}{2} = \frac{0.05}{\ln(1.2)} \approx 0.03 \text{ sec from pt} \]

\[ \frac{3}{2} = \frac{0.05}{\ln(1.2)} \approx 0.03 \text{ sec from pt} \]

\[ \frac{3}{2} = \frac{0.05}{\ln(1.2)} \approx 0.03 \text{ sec from pt} \]

\[ \text{Real part of dominant pole} = -\frac{5}{2} \approx -2.5 \]

\[ \text{Imaginary part} = \frac{\sqrt{5}}{2} \approx 1.2 \]

\[ -5(2.31) = -11.55 \] 25. The pole, $s = -200$, is OK, but the zero $(5.354, 280)$ is close to the pole.
ALTERNATE APPROACH USING $T_S$, SETTLING TIME

1. $C(s) = 2 - (s/0.02) = 1.96$ FROM Plot, $T = 0.093$ sec

   $T_S \approx \frac{A}{\sum \mu} = 0.093, \ \mu = \frac{A}{C_{ss}} = \frac{2}{0.026(0.093)} = 19.96 \text{ milsec}$

2. Overlap test: $\frac{\text{Num. Pole Real Part}}{\text{Gain}} = \frac{5}{1.96 \times 19.96} = -43.01$ Complex part $= \pm \sqrt{1^2 - (-43.01)} = \pm 19.95$

   NOT AND OTHER POLE AND ZER REAL PARTS MUST BE

   $\leq (5)(-43.01) = -215.05$. THE POLE AT -200 IS OK, BUT THE ZER AT $s = -52.3$ IS TOO CLOSE TO THE DOMINANT POLE.
a) Convert the following system block diagram to a Signal-Flow-Graph and b) find \( C(s)/R(s) = G_{CL}(s) \) using Mason's Rule.

\[
\alpha = -G_2 H_2
\]

Transmission Path \( s = \alpha \)

\[
T_1 = G_1 G_2 > T_2 = G_2 G_3
\]

\[
\Delta = 1 - \alpha
\]

\[
\begin{align*}
\text{at } T_1 & : D_1 = 1 \\
\text{at } T_2 & : D_2 = 1
\end{align*}
\]

\[
\frac{C(s)}{R(s)} = G_{CL}(s) = \frac{T_1 + T_2}{\Delta}
\]