Exercise 3.8

\[ M \ddot{x} + c \dot{x} + kx = ky \]

Let \( x = \text{Re} \left( \frac{Y}{i} \exp(i\omega t) \right) \)

\[ y = \text{Re} \left( \frac{Y}{i} \exp(i\omega t) \right) \]

So

\[ x = \frac{kY}{k + i\omega c - \omega^2 M} \]

\[ x = \text{Re} \left[ \frac{Y}{i} \frac{k}{k + i\omega c - \omega^2 M} \exp(i\omega t) \right] \]

\[ F = k(y - x) = \text{Re} \left[ \frac{Y}{i} \left( 1 - \frac{k}{k + i\omega c - \omega^2 M} \right) \exp(i\omega t) \right] \]

\[ = \text{Re} \left[ \frac{Y}{i} \frac{i\omega c - \omega^2 M}{k + i\omega c - \omega^2 M} \exp(i\omega t) \right] \]

Rewrite \( x \) as

\[ x = \text{Re} \left[ \frac{Y}{i} \frac{k/M}{k/M + i\omega c/M - \omega^2} \exp(i\omega t) \right] \]

\[ = \text{Re} \left[ \frac{Y}{i} \frac{1}{1 + 2i\sqrt{r} - r^2} \exp(i\omega t) \right], \quad r = 1.1 \]

Thus

\[ |x| = |Y| \left| \frac{1}{1 + 2i\sqrt{r} - r^2} \right| \]

Set \( r = 1.10 \quad \text{and} \quad g = 0.2 \Rightarrow |x| = 2.05 |Y| \]

\[ \text{arg}(X) = \text{arg}(Y) - \phi \]

\[ = \text{arg}(Y) - \tan^{-1} \left( \frac{2g}{1 - r^2} \right) \]

\[ = \text{arg}(Y) - 2.935 \quad \text{rad} \]
Exercise 3.19

\[ \dot{q} = \frac{\dot{V}}{k} \]

\[ Q(t) = \frac{\epsilon m \pi^2}{\sqrt{2}} \sin(nt) \]
\[ = \text{Re} \left\{ \frac{1}{i} \frac{\epsilon m \pi^2}{\sqrt{2}} \exp(i nt) \right\} \]

Let \( q = \text{Re} \left\{ \frac{1}{i} \frac{\epsilon m \pi^2}{\sqrt{2}} \exp(i nt) \right\} \)
\[ = |y| \sin(nt - \phi) \]
\[ \gamma = \frac{Em}{M+m} \frac{r^2}{1-r^2+2iyr} \]

Set \( M+m = 80 \text{ kg} \), \( k = \frac{(M+m)\gamma}{\delta} = \frac{80(9.807)}{0.04} \text{ N/m} \)
and \( \delta = 0.005 \text{ meter} \)

When \( \phi = 145 \frac{2\pi}{360} \) rad/s, have \( |y| = 0.01 \text{ meter} \)
and \( q = 0 \) when \( nt = 75 \left( \frac{2\pi}{360} \right) \)
But \( q = |y| \sin(nt - \phi) \Rightarrow q = 0 \) when \( nt = \phi \)

Thus \( \phi = 75 \left( \frac{2\pi}{360} \right) \) rad \( \Rightarrow \phi = 145 \left( \frac{2\pi}{60} \right) \)

Evaluate
\[ \omega_{act} = \left( \frac{k}{M+m} \right)^{1/2} = (\frac{\delta}{\gamma})^{1/2} = 15.658 \text{ rad/s} \]

Then \( \phi = 145 \left( \frac{2\pi}{60} \right) \Rightarrow r = \frac{151.84}{15.658} \)

Find \( y \) from phase angle:
\[ \frac{2.89 \frac{r}{1-r^2}}{} = \tan \left( \frac{150\phi}{360} \right) \Rightarrow \delta = \frac{\frac{r^2}{(\frac{r}{1-r^2})}}{2n} \text{ tan} \left( 1,309 \right) = 0.11467 \]

Find \( \epsilon m \) from amplitude:
\[ \epsilon m = |y| (M+m) (1-r^2+2iyr) / r^2 = 0.19568 \text{ kg-m} \]

When \( r > \text{what} \), \( \min |y| \) corresponds to \( r \to 0 \), so
\[ \min |y| = \frac{\epsilon m}{M+m} = 0.002498 \text{ meter} \]
Exercise 3.22

Given imbalanced mass (call it \( m_3 \)) at distance \( r \), spinning tub \( m_1 \), @ radius \( R \), framework mass \( m_2 \), \( \omega_{at} = 10 \pi \) rad/s, \( \gamma = 0.1 \)

Find eq of motion, smallest amplitude if \( R > \omega_{at} \)
largest amplitude & corresponding \( \Omega \).

Solution: Form \( T \) corresponding to situation where \( \Omega = 0 \) \( \Rightarrow \) translation in \( x \) direction:

\[
T = \frac{1}{2} (m_1 + m_2 + m_3) \dot{x}^2 \Rightarrow M_{11} = m_1 + m_2 + m_3
\]

\[
V = \frac{1}{2} k x^2 \Rightarrow K_{11} = 2k
\]

\[
\dot{\rho}_{13} = (m_3 \dot{x}^2) \Rightarrow C_{11} = 2M
\]

The unbalanced mass is the ball of clothing \( m_3 \), whose center of mass is at distance \( r \), so the excitation is a force \( m_3 r R^2 \). If \( t = 0 \) is defined as the instant when the ball is vertical, then the force component in the direction of \( x \) is \( (m_3 r R^2) \cos(\Omega t) \). Thus

\[
(m_1 + m_2 + m_3) \ddot{x} + 2M \dot{x} + 2kx = m_3 r R^2 \cos(\Omega t)
\]

Note that the fact that the tub rotates is irrelevant. Divide by the mass coefficient:

\[
\ddot{x} + 2 \gamma \omega_{at} \dot{x} + \omega_{at}^2 x = r \frac{m_3}{m_1 + m_2 + m_3} R^2 \Re(e^{i\Omega t})
\]

Let \( x = \Re(Xe^{i\gamma t}) \) & cancel \( e^{i\gamma t} \) factor:

\[
\ddot{X} = r \frac{m_3}{m_1 + m_2 + m_3} \frac{\Omega^2}{\omega_{at}^2 + 2 \gamma \omega_{at} R - \Omega^2}
\]

Note \( |x| = \text{amplitude}, \omega_{at} = \frac{2k}{(m_1 + m_2 + m_3)}, \gamma = \frac{2 \mu}{(2 \omega_{at} M_{11})} \)