Electromechanical Systems

Modeling and Simulation in the Frequency Domain

- **Transfer Functions, Electromechanical Systems**. This is a PowerPoint presentation illustrating an electromechanically system. A motor corresponding load. Transfer functions are automatically obtained and the Bode Plots (Frequency Response Plots) are discussed.

Modeling Difficulties

- **Systems with dependent elements (Derivative Causality)**. This is an in depth discussion on what does physically mean when a system has elements in derivative form. What are the possible solutions for problems with derivative causality. What do the solutions mean in terms of physical elements that modify systems.

- **Systems with Algebraic Loops**. This is a discussion of another modeling difficulty which is the algebraic loops. This means that depending on how you connect elements, you can cause it to have derivative form which always is a major challenge for computational purposes.

Transfer Functions and State Space Models

- From the previous study guide, you were asked to study the concept of State Space form of the differential equations of a mechatronics system. You must know how to obtain the A, B, C, D matrices. Then perform simulations based on the State Space Form.

  - You also need to know how to obtain the transfer functions knowing the state space form. I will explain this in class, but if you are having difficulty understanding this, please come and see me.

Assignments

1. **Homework**. (Due Tuesday, April 21, 2014, 11:59 p.m.)
   Please do the following problems from your textbook 5-1, 5-4, 5-8, 5-9, 5-10, 5-16.
   Please scan, transfer them over and keep your originals.

2. **Computer Assignment**. (Due on Tuesday, April 25, 2014, 11:59 p.m)
   Please do not send your assignments via email, except on emergencies

3. **Quiz**. Tuesday April 29, 2014
MODELING AND SIMULATION OF PIEZOELECTRIC SENSORS.

1.1 PIEZOELECTRIC SENSORS

Piezoelectric sensors are used in many engineering applications to measure physical systems variables. The device such as the one shown in Figure 1.1 consists of an oscillating mass with damping and stiffness effects enclosed in housing. The force acting on the mass is transmitted to a piezoelectric material in the form of a wafer attached to the oscillating mass. When such material is loaded, produces an electric charge associated with the mechanical deformation. Since such a charge is small, it requires an operational amplifier in order to be measured and find a relation to the acceleration, which is trying to measure.

![Piezoelectric Sensors](image)

**Fig 1.1 Piezoelectric Sensors**

There are essentially three parts to the sensor. One is the Mechanical part, the second is the piezoelectric transformation and the last one is the electronics necessary to produce the amplification. The oscillating mass is a second order mass, damper, stiffness system.

1.2 Operational Amplifier Model

![Operational Amplifier Model](image)
For the reader not familiar with bond graphs, it may be useful to make the equivalence of the equations that control the operation of the amplifier obtained from basic physical principles and those using the Bond Graph Model notation. Basic operation of the Operational Amplifier requires that the input current be zero. In order to achieve that, the operational amplifier has to have very high input impedance. U2 (Fig 1.2a) is small voltage and in an ideal case the whole current \( i_q \) flows through the resistor and the capacitor. The current \( i_q \) can be expressed:

\[
i_q = -i_2
\]

and thus

\[
i_q = \frac{U_a}{R} \frac{dU_a}{dt} C
\]  

Which leads to the differential equation in terms of the output voltage.

\[
RC \frac{dU_a}{dt} + \frac{U_a}{R} = -R \frac{dq}{dt}
\]

The Figure 1.2b represents the bond graph model of the amplifier. The amplifier gain is represented by a controlled source whose input is only the voltage signal U2 and its voltage amplification is U2*A. Since Ua is a high voltage compared to U2, it follows that the current flows as indicated through the R and the C elements. The zero junctions are the points of common potential and they have been laid out to resemble the physical layout of the circuit. The voltage across the R and C elements is the difference between Ua and U2 and the 1 junction represents such. Both the R and C elements are attached to the 0 junction to indicate the common potential across them. This also indicates that the current i2 is the sum of the currents through C and R and thus ic+ir=i2. The arrows in the 0 and 1 junctions indicate how the currents or the voltages sum or subtract.

Figure 1.3 Indicates that an augmented and simplified Bond Graph where the variables have been generalized as R, C, and SE elements referenced by their bond number and the type of element they represent. Such notation allows the description of a physical model by a bond graph into computer software like CAMP-G in such generalized form so that systems in different energy domains can be processed. The e variables are voltages and the f variables are currents in this case.
Using the bond graph notation above, junctions are numbered and the physical parameters are distinguished by the bond number to which they are attached. Using the 0 and 1 connections together with the individual element constitutive relations, the differential equation can be derived in terms of the capacitor charge \( q \) as a state variable. The variable chosen here is the charge as a state because in fact it is the current \( i_2 \) that gets integrated. The corresponding differential equation \( dq_{12}/dt \) is written as \( dq_{12} \) and the voltage output equation is \( e_{11} \) or \( e_{12} \):

\[
\begin{align*}
\frac{dQ_{12}}{dt} &= -\frac{SF_6}{C_{12}/R_{13}} \quad (1.3) \\
e_{11} &= Q_{12}/C_{12}
\end{align*}
\]

Here one can clearly recognize that \( f_7 = -f_6 \), which in fact is \( i_q = -i_2 \) in the schematic of Fig 3. Also \( f_{11} = f_7 = -f_6 = -i_q \) and \( f_{12} = f_{11} - f_{13} \)

\( f_6 \) is \( i_q \), \( f_{12} = i_C \) and \( f_{13} = i_R \). Now it can be demonstrated that these equations have a direct relation to those used with the conventional notation.

\( f_{12} = f_{11} - f_{13} \) is \( i_C = i_q - i_R \) and thus

The following equation is obtained

\[
f_{12} = -\frac{dU_a}{dt} C
\]

\[
-\frac{dU_a}{dt} C = i_q - \frac{U_a}{R} \quad \text{And thus yields} \quad (1.5)
\]

\[
RC\frac{dU_a}{dt} + \frac{U_a}{R} = -R \frac{dq}{dt} \quad \text{The same as equation (1.2)} \quad (1.6)
\]

Hoffman J, (1998) also obtained and equation like (1.6) applying basic principles. This verifies the validity and complete equivalence of the Bond Graph representation of the Operational Amplifier. One should notice that the choice of state variable \( q \) in this case (charge) is automatically made to be the result of the physical variable that gets integrated (current \( i \)). One can take the voltage \( U_a \) as state but a lot of manipulation of the equations is necessary such as shown in Hoffman J. (1998).

1.3 Piezoelectric Transformation

The piezoelectric transformation is a direct relation between the displacement of the mass and the charge generated in the capacitor so that yields.

\[
q = K_q y \quad \text{And thus} \quad (1.7)
\]

\[
\frac{dq}{dt} = K_q \frac{dy}{dt} \quad (1.8)
\]

Which represents the piezoelectric transformation between the velocity of the oscillating mass and the current, which is the input to the operational amplifier. This equation in Bond Graph notation is represented by a transformer element like:

\[
\begin{array}{c}
\text{Mechanical} \\
\text{Side}
\end{array}
\quad \text{TF}
\quad \begin{array}{c}
\text{Electronic} \\
\text{Side}
\end{array}
\]

\[
\begin{array}{c}
y \quad \text{TF} \quad q
\end{array}
\]
1.4 Mechanical Model

The mechanical model consists of the housing and a mechanical oscillator with stiffness and
damping. The physical system and Bond Graph are shown in Fig 1.5.

Here the velocities \( y \) and \( u \) are absolute velocities with respect to an inertial frame.
Considering that the piezoelectric effect is produced by the relative displacement of the mass on
the piezoelectric material then it makes sense to express the Bond Graph and indeed the equations
of motion of the mass in terms of the relative motion with respect to the housing. This means a
simplification of the model without considering the mass of the housing, but considering that the
housing transmits acceleration to the mass. The following Bond Graph represents this. The
physical parameters and expressions for the forces have been superimposed on the bond graph for
clarity, but they are not required as the bond graph uses the effort variables and the bond numbers
to express the generalized forces. The forces superimposed on the bond graph of Fig 1.6b will
help the reader not familiar with Bond Graphs to demonstrate the equivalence of the differential
equations that one can obtain using the free body diagram. The 1 junction represents the
summation of forces in the same way.

\[
m \ddot{y} + b \dot{y} + ky = -m \ddot{u}
\]  
(1.9)

Now it is necessary to assemble the Piezoelectric Sensor as a complete system.
YOUR LAB ASSIGNMENT

Objective: Obtain State Space Equations Form, Obtain a transfer function manually and a computer generated Transfer functions. Obtain a Frequency response simulation to determine the frequencies in which the sensor will measure acceleration accurately and in which frequency range it will not.

1.- Assemble and complete the electromechanical model using the bond graph modeling technique. Indicate the components and connections.
2.- Enter Bond Graph model in CAMPG
3. Generate Differential Equations in Cauchy form
4. Obtain the computer generated transfer functions
   o A) Symbolically
   o B) Numerically
5. In MATLAB. Using the computer generated transfer function obtain:
   • The step response
   • The impulse response
7. Perform a simulation Using the state Space form produced by CAMPG in Simulink. This means transform the state space representation of the system into a block diagram and perform the simulation to compare the results. Demonstrate they are the same.
8. Perform a simulation using the computer generated Transfer Function in Simulink

WHAT TO TURN IN:

Please turn all these questions summarized on a Word Document along with the Bond Graph Files, the Matlab files, Simulink files and the results.

Please turn in a directory by creating and transferring files that contain:
   - A PowerPoint or Word Files explaining and describing what you did and the steps you follow to solve the problem described. You may want to take screen shots of the work and paste them in your document as you go on.
   - The CAMPG (.bg), MATLAB (.m), SIMULINK (.mdl) files used

Name your directory Yourlastname_ME270S13_Sensor

Please turn electronically to the path indicated on Voyager ...\