Problem 1

We want the planar RRR manipulator shown in figure 1 to grasp the tennis ball on the floor and put it on the table as shown in the figure. This is a path planning problem that requires an inverse kinematic solution first. The lengths of the links are:

\[ a_1 = 30\text{cm} \]

\[ a_2 = 30\text{cm} \]

\[ a_3 = 10\text{cm} \]

We want the manipulator to grasp the ball vertically (this adds a constraint on the robot orientation). The initial location of the ball has coordinates \((-20, 0, 0)\) in the base reference frame, and its desired “location” on the table has coordinates \((50, 20, 0)\). The orientation of the end effector is denoted by \(\beta\), and the position by \(p_x, p_y\). The robot’s home position is shown in figure 3 (third from left) and characterized by

\[ \theta_1 = 90^\circ \]

\[ \theta_2 = 0^\circ \]

\[ \theta_3 = 0^\circ \]

It is clear that \(0 \leq \theta_1 \leq 180^\circ\). This constraint must be always satisfied. A path that requires a value of \(\theta_1\) outside of this interval is not feasible and therefore cannot be used. Figure 2 shows an example of the manipulator’s path and the corresponding time evolution of the configuration variables.

1) Write the equations that give the location \((p_x, p_y)\) and orientation \((\beta)\) of the end effector as a function of the configuration variables.

2) What is the position and orientation of the end effector at the home configuration?

3) Determine the robot configuration allowing to put the end effector at a desired position and orientation given by

\[ \begin{bmatrix} p_x \\ p_y \\ \beta \end{bmatrix} = \begin{bmatrix} -20\text{cm} \\ 0\text{cm} \\ -90^\circ \end{bmatrix} \]

4) Determine the robot configuration allowing to put the end effector at a desired position and orientation given by

\[ \begin{bmatrix} p_x \\ p_y \\ \beta \end{bmatrix} = \begin{bmatrix} 50\text{cm} \\ 20\text{cm} \\ -90^\circ \end{bmatrix} \]

5) Write code for path planning using the independent joint proportional control method to move the robot from its home configuration to the configuration given by (7). Make sure your proposed
Fig. 1. 3-link planar manipulator

Fig. 2. Example of path planning from configuration [150°, 20°, 0°] to the home position and time evolution of the configuration angles

path is feasible. Show your code and the robot’s path (An example of the robot’s path is shown in figure 2–left).

6) Plot the time evolution for the configuration variables (An example of the time evolution of the configuration variables is shown in figure 2–right).

Problem 2

Figure 4 shows the robot of interest which is a popular RRR manipulator with

\[ d_1 = 50\text{cm} \]  \hspace{1cm} (9)
\[ d_2 = 40\text{cm} \]  \hspace{1cm} (10)
\[ d_3 = 15\text{cm} \]  \hspace{1cm} (11)
1) Write the homogenous transformations $H_{01}^1$, $H_{12}^2$, and $H_{23}^3$.

2) Based on the homogenous transformations, determine the coordinates of the end effector when

$$\begin{bmatrix}
\theta_1 = \pi/2 \\
\theta_2 = \pi/2 \\
\theta_3 = 0
\end{bmatrix}$$

(12)

No need for an analytic solution for $H_{03}^0$.

3) Based on the homogenous transformations, determine the orientation of the end effector when

$$\begin{bmatrix}
\theta_1 = \pi/2 \\
\theta_2 = \pi/2 \\
\theta_3 = 0
\end{bmatrix}$$

(13)

4) Calculate the configuration variables to place the end effector at position

$$\begin{bmatrix}
25cm \\
25cm \\
15cm
\end{bmatrix}$$

(14)

Problem 3

A robotic tennis player is shown in figure 5. The robot consists of two revolute and one prismatic joint that translates in the z-direction. The robot is configured to hit the ball at $\theta_1 = \pi/2$, which corresponds to the plane $x_0 = 0$. The lengths of the links are $d_1 = 1.5m$, $d_2 = 0.5m$. 
1) Write the position of the Tennis racket in the base reference frame as a function of the configuration variables. Keep the equations as simple as possible.

2) What is the location and orientation of the tennis racket at configuration

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} -\pi/4 \\ 5\pi/6 \\ 0.3m \end{bmatrix} \quad (15)$$

3) Assuming that the rendezvous point (where the robot hits the ball) is

$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5657m \\ 0.9343m \end{bmatrix} \quad (16)$$

Calculate the corresponding robot configuration.

4) We are interested in the upper part of the robot Jacobian, that is

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \end{bmatrix} \quad (17)$$

Find an expression for matrix $J$.

5) Calculate the numerical values of $J$ at the rendezvous point.

6) What are the joint speeds allowing to hit the ball with speed

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} 1m/s \\ 0.2m/s \\ 0.5m/s \end{bmatrix} \quad (18)$$

7) We want to plan the robot path using the polynomial trajectories independent joint control method. Since we want to hit the ball with a predetermined speed, a cubic polynomial is needed.
Determine the path parameters $a_i, i = 1, ..., 4$ for each of the joints knowing that

$$\begin{bmatrix}
\theta_1(0) = \pi \text{rad} & \theta_2(0) = 0 \text{rad} & d_3(0) = 0.1 \text{m} \\
\dot{\theta}_1(0) = 0 \text{rad/s} & \dot{\theta}_2(t_0) = 0 \text{rad/s} & \dot{d}_3(0) = 0 \text{m/s} \\
\theta_1(t_f) = \pi/2 \text{rad} & \theta_2(t_f) = -0.7854 \text{rad} & \dot{d}_3(t_f) = 0.3 \text{m} \\
\dot{\theta}_1(t_f) = -1.7677 \text{rad/s} & \dot{\theta}_2(t_f) = 0.6187 \text{rad/s} & \dot{d}_3(t_f) = -0.2121 \text{m/s}
\end{bmatrix} \quad (19)$$

The initial time is 0 and the final time is 2s.

8) Plot the time evolution of the joint variables. An example is shown for $\theta_1$ in figure 6–left.
9) Plot the time evolution of the joint speeds. An example is shown for $\dot{\theta}_1$ in figure 6–right.
10) Use Matlab or similar tools to show the manipulator at the rendezvous point. Links are represented by lines and joints by circles, an example is shown in figure 7.