Problem 1

A linear system is characterized by the transfer function

\[ \frac{Y(z)}{U(z)} = \frac{0.5z + 0.35}{(z - 0.5)} \]  

1) Find the time response \( y(k) \) to a unit impulse input.
2) Find the time response \( y(k) \) to a unit step input.
3) For a unit step input, find \( y(\infty) \) using the final value theorem.
4) For a unit step input, find \( y(\infty) \) using the time sequence.

Problem 2

For a unity feedback system, we are given the analog subsystem

\[ G(s) = \frac{s}{(s + 5)} \]  

The system is digitally controlled with a sampling period of 0.02s. The controller transfer function was selected as

\[ C(z) = \frac{0.35}{(z - 1)} \]  

1) Find the z-domain transfer function \( G_{za} \) for the analog subsystem with DAC and ADC.
2) Find the closed loop transfer function and the characteristic equation.
3) Find the steady state error for a sampled unit step.
4) Find the steady state error for a sampled unit ramp.
5) Discuss the effect of the controller on the steady state error.

Problem 3

Consider the open loop transfer function

\[ L(z) = \frac{1}{(z - 1)(z - 0.8)} \]  

Assume a unity-feedback configuration with DC gain.
1) Write the closed loop characteristic equation.
2) Use the Jury test to check for the stability of the system in terms of the gain.
3) Design a proportional controller with sampling period of \( T = 0.1s \) to obtain
   a) A steady state error of 10%. Use appropriate error constant.
   b) A damped natural frequency \( \omega_d \) of 3 rad/s.
4) Are the controllers you designed in questions (a) and (b) realizable?

Problem 4

Consider a system with transfer function

\[ L(z) = \frac{0.5}{z - 0.75} \]  

We want to design a proportional control with unity feedback. The sampling time is 0.1s. Find the range of values of \( K_P \) such that the closed-loop system
1) Is stable
2) Has steady-state error to a unit step reference less than 0.1
3) Has setting time \( T_s < 10 \)
4) Has maximum overshoot \( M_P < 0.1 \)

(Note: It may not be possible to satisfy all criteria.) Repeat with one time delay in the feedback loop. Explain the change, if any, in \( K_P \).