Basic Concept:

Add 2 voltages, one with a $-\text{T}_{\text{CF}}$, and one with a $+\text{T}_{\text{CF}}$, to get a sum with 0 $\text{T}_{\text{CF}}$.

(Same idea as with Zener reference!)

Here, use $V_{\text{BE(ON)}}$ to get $A - \text{T}_{\text{CF}}$, and use $V_k = \frac{\Delta T}{T}$ to get $A + T^0$.

(e.g., from a $\Delta V_{\text{BE}}$ CT)

Then,$V_o = V_{\text{BE(ON)}} + K V_k$

Typical results show $\frac{\Delta V_{\text{OUT}}}{\Delta T}$ at a particular temperature, and only small variation over a wide range.

However, some residual curvature remains due to second order effects.

For band-gaps, $\text{T}_{\text{CF}}$ is typically defined over a broad range by:

$\text{T}_{\text{CF}} \text{ "effective" } = \frac{1}{V_{\text{OUT}}} \left( \frac{V_{\text{OMAX}} - V_0(\text{MIN})}{T(\text{MIN}) - T(\text{MAX})} \right)$

Typical "good" #s: $< 50^\circ \text{C}$.
To find $K$, we need to know the TCF of $\text{Vbe(on)}$ accurately.

1st, $V_{\text{be(on)}} = V_x \ln \left( \frac{I_s}{I_t} \right)$, where $V_x = \frac{ET}{q}$.

And, $I_s = \frac{qA_m^2\mu_m}{Q_B} = \frac{qA_m^2(\frac{ET}{q})\mu_m}{Q_B}$

Using the Einstein relation $P_m = \frac{ET}{\mu_m}$

$\Rightarrow I_s = B\mu_m^2T\mu_m$, were $B = A$ constant w.r.t. temperature

Now, $\mu_m$ can be written as: $\mu_m = C T^{-n}$, where $C = $ another constant w.r.t. $T$

Also,

$\mu_m^2 = DT^{3/2}(\frac{V_{ce}}{V_x})$.

Where:

$D = $ another constant w.r.t. $T$

$V_{ce} = $ the band-gap voltage of silicon (at $0^\circ K$)

---

Fig. 3.6 Temperature dependence of (a) electron and (b) hole mobilities in silicon for specific dopings, ranging from $10^{17}/\text{cm}^3$ to $10^{20}/\text{cm}^3$. (From A. B. Phillips, Transistors, Estimation and Introduction to Semiconductors, 2012.)
\[ I_s = B m \alpha^2 T \bar{I}_m \]
\[ = B \left( D T^3 \frac{e^{(-\frac{V_{BE}}{V_T})}}{e^{(-\frac{V_{BE}}{V_T})}} \right) T (C T^{-m}) \]
\[ I_s = (B \cdot D \cdot C) (T^{-m}) \frac{e^{(-\frac{V_{BE}}{V_T})}}{e^{(-\frac{V_{BE}}{V_T})}} \]

**SUBSTITUTE THIS INTO:**

\[ V_{BE}(m) = V_T \ln \left( \frac{I_s}{I_s} \right) \]

\[ \Rightarrow V_{BE}(m) = V_T \ln \left[ (\frac{I_s}{I_s}) \left( e^{T^{-m}} \frac{e^{(-\frac{V_{BE}}{V_T})}}{e^{(-\frac{V_{BE}}{V_T})}} \right) \right] \]

**WHERE:** \( y = 4 - m \), AND \( E = \frac{1}{B \cdot D \cdot C} \) = ANOTHER CONSTANT W.R.I.T. \( T \)

**NOW** ASSUME THAT \( I_c(T) = G T^\alpha \) IN THE BAND-GAP CIRCUIT

\[ \text{WHERE: } G = \text{ANOTHER CONSTANT W.R.I.T. } T \]

\[ \Rightarrow V_{BE}(m) = V_T \ln \left[ E G T^{\alpha - y} \right] + V_T \ln \left[ e^{(-\frac{V_{BE}}{V_T})} \right] \]

\[ = V_{BE} + V_T \left[ \ln(E G) + \ln(T^{\alpha - y}) \right] \]

\[ V_{BE}(m) = V_{BE} - V_T \left[ (\alpha - y) \ln T - \ln(E G) \right] \]

**NOW** RECALL \( V_{OUT} = V_{BE}(m) + K V_T \) FOR THE BAND-GAP, WHERE \( K = A \text{ MULTIPLIER TO BE CHOSEN TO SET } T_C = 0 \)

\[ \Rightarrow V_{OUT} = V_{BE} - V_T (\alpha - y) \ln T + V_T (K + \ln(E G)) \]

**NOTE:** \( K, G, \alpha \) ARE CIRCUIT PARAMETERS & \( E, y \) ARE DEVICE PARAMETERS
\[
\frac{\partial V_{\text{out}}}{\partial T} = \frac{-V_t}{T} (y-\alpha) \ln T - \frac{V_t}{T} (y-\alpha) + \frac{V_t}{T} (k + \ln E_G)
\]

If \( \frac{\partial V_{\text{out}}}{\partial T} = 0 \) @ \( T = T_0 \),

\[
\frac{V_t}{T_0} (y-\alpha) [\ln T_0 + 1] = \frac{V_t}{T_0} (k + \ln E_G)
\]

\[
(k + \ln E_G) = (y-\alpha) [\ln T_0 + 1]
\]

or,

\[
k = (y-\alpha) [\ln T_0 + 1] - \ln E_G
\]

If the last form is substituted back into the equation for \( V_{\text{out}} \),

\[
V_{\text{out}} = V_{\text{ao}} - \frac{V_t}{T} (y-\alpha) \ln T + \frac{V_t}{T} (y-\alpha) [\ln T_0 + 1]
\]

\[
= V_{\text{ao}} + \frac{V_t}{T} (y-\alpha) [1 + \ln T_0 - \ln T]
\]

\[
V_{\text{out}} = V_{\text{ao}} + \frac{V_t}{T} (y-\alpha) [1 + \ln \frac{T_0}{T}]
\]

Thus, the temperature dependence of \( V_{\text{out}} \) is completely characterized by \( T_0 \). The temperature at which \( \frac{\partial V_{\text{out}}}{\partial T} = 0 \) (different \( k \) = different \( T_0 \)).

(Note that \( y \) is a device parameter & \( \alpha \) is set by the circuit used)

If a "PTAT" current source (e.g., a \( \Delta VBE \) circuit) is used \( \Rightarrow \alpha = 1 \)

("PTAT" = "proportional to absolute temperature")
EXAMPLE

A band-gap reference is designed to give a nominal output voltage of 1.205 V, which gives zero $T_C$, at 25°C. Because of component variations, the actual room temperature output voltage is 1.280 V. Find the temperature of actual zero $T_C$ of $V_{\text{out}}$ and $V_{\text{out}}(T)$, and calculate the $T_C$ at room temperature. Assume that $\gamma = 3.2$ and $a = 1$. Since the behavior of the output voltage is characterized by $T_0$, we first calculate the value of $T_0$. From (4.191),

$$V_{\text{out}}(T) = V_{\text{out},0} + V_T(\gamma - a)(1 + \ln \frac{T}{T_0})$$

(4.193)

At 25°C,

$$1.280 V = 1.205 V + (26 \text{ mV})(2.2)(1 + \ln \frac{T_0}{300^oK})$$

and thus

$$T_0 = 300^oK \left( \frac{\exp \frac{16 \text{ mV}}{57 \text{ mV}}}{\exp \frac{15.6 \text{ mV}}{57 \text{ mV}}} \right) = 411^oK.$$

This is the temperature where $T_C$ of $V_{\text{out}}$ will be zero. Thus we can express $V_{\text{out}}$ as

$$V_{\text{out}}(T) = 1.205 + 57 \text{ mV}(1 + \ln \frac{411^oK}{T})$$

Differentiating (4.191),

$$\frac{dV_{\text{out}}}{dT} = \frac{1}{T} \left[ V_T(\gamma - a)(1 + \ln \frac{T}{T_0}) \right] - \frac{V_T}{T} (\gamma - a) \ln \frac{T}{T_0}$$

If $T$ is near $T_0$, then

$$\ln \frac{T}{T_0} = 1 \ln (\frac{T_0 - T}{T}) = \frac{T_0 - T}{T}$$

and we have

$$\frac{dV_{\text{out}}}{dT} = \frac{V_T}{T} \left( \frac{T_0 - T}{T} \right) (\gamma - a)$$

For $T_0 = 411^oK$, $T = 300^oK$,

$$\frac{dV_{\text{out}}}{dT} = \frac{26 \text{ mV}}{300^oK} \left( \frac{411 - 300}{300} \right)(2.2) = 70 \mu V/^oC.$$

This is the $T_C$ of $V_{\text{out}}$ at room temperature.
TO UNDERSTAND THIS CIRCUIT, FIRST EXAMINE SUBCIRCUIT AT RIGHT

- As $V_1$ increases from $0$, $Q_1$ and $Q_2$ remain off until $V_1 > V_{BE(on)}$, so $I_{C1} = I_{C2} = 0$ and $V_2 = V_1$. They turn on at $\theta$.

- When $Q_1, Q_2$ first turn on, their $I_C$ are small, so the drop across $R_3$ is negligible, so $I_{C1} = I_{C2}$. Since $R_2$ is typically $\gg R_1$, $Q_2$ saturates (Point $\alpha$).

- Because of $R_3$, $I_{C2}$ increases slowly ($\phi_1$), $V_2$ decreases slowly ($\phi_2$). As $V_1$ increases, $Q_2$ comes out of saturation (Point $\beta$).

- Further increases in $V_1$ would be reflected in $V_2$, as shown. However, $Q_3$ goes out of operation at $V_2 = V_{BE(on)}$ (Point $\gamma$).

(Note that since more than 1 stable operating point exists, $\theta_1 + \phi_2$, a start-upckt is needed!)

Now, in stable operation, $V_{OUT} = V_{BE(on)} + V_{R2}$

Also, since $I_{C2} \approx I_{C1}$, $V_{R2} = V_{R3} \left( \frac{R_{2}}{R_{3}} \right)$

where: $V_{R3} = \Delta V_{BE} = V_T \ln \left( \frac{I_1}{I_{S1}} \right) - V_T \ln \left( \frac{I_2}{I_{S2}} \right)$

Since $I_1 = I_2 \left( \frac{R_{2}}{R_{1}} \right)$,$\frac{V_{OUT} = V_{BE(on)} + \left( \frac{R_2}{R_1} \right) V_T \ln \left( \frac{R_2}{R_1} \cdot \frac{I_{S2}}{I_{S1}} \right) }{R_1}$
**Improved Ckts**

**Figure 4.36c** improved band gap reference.

**Better Ckt:**
Here, the op amp forces

$$V_{R2} = V_{R1} \Rightarrow I_2 = I_1 \left( \frac{R_1}{R_2} \right)$$

And,

$$V_{out} = V_{BE(ON)} + I_1 R_1$$

Since

$$V_{R3} = \Delta V_{BE} = V_{X} + \frac{1}{n} \left( I_1 R_1 \right)$$

$$= \left[ V_{BE(ON)} + \frac{R_2}{R_1} \left( \frac{V_{X} + \frac{1}{n} \left( I_1 R_1 \right)}{R_1} \right) \right]$$

$$= V_{BE} + k V_{X}$$, as desired.

**Brokaw Cell (1974)**
Figure 12.20 Example of a $V_{BE}$-referenced self-biased reference circuit.

Fig. 1. Basic bandgap configurations: (a) with positive output respect to ground; and (b) with negative output respect to the positive supply.
\[
\Delta V_{BE} = V_{BE2} - V_{BE1} = V_T \ln \left( \frac{I_2}{I_{S2}} \right) - V_T \ln \left( \frac{I_1}{I_{S1}} \right)
\]
\[
\Delta V_{BE} = V_T \ln \left[ \left( \frac{I_2}{I_{S2}} \right) \left( \frac{I_{S1}}{I_1} \right) \right]
\]

**Also, the op amp forces**  
\( I_1 R_3 = I_2 R_4 \)  
\( \Rightarrow I_2 = \frac{R_3}{R_4} I_1 \)

\[
\Rightarrow \Delta V_{BE} = V_T \ln \left[ \left( \frac{R_3}{R_4} \right) \left( \frac{I_{S1}}{I_1} \right) \right]
\]

**Now,**  
\( V_0 = \left(1 + \frac{R_3}{R_C} \right) \left( V_{BE2} + V_{R1} \right) \)

**Where:**  
\( V_{R1} = (I_1 + I_2) R_1 = I_1 \left(1 + \frac{R_3}{R_4} \right) R_1 = I_1 R_2 \left( \frac{R_1}{R_2} \right) \left(1 + \frac{R_3}{R_4} \right) \)

**Assuming**  
\( \alpha \approx 1 \)

\[
\Rightarrow V_0 = \left(1 + \frac{R_3}{R_C} \right) \left( V_{BE2} + V_T \left( \frac{R_1}{R_2} \right) \left(1 + \frac{R_3}{R_4} \right) \ln \left[ \frac{R_T}{R_4} \left( \frac{I_{S1}}{I_1} \right) \right] \right)
\]

**If:**  
\( R_3 = R_4 \)  
\( I_1 = I_2 \)  
\( I_{S1} = \alpha I_{S2} \)  
\( K = \frac{2}{R_2} \ln(\alpha) \)

\[
V_0 = \left(1 + \frac{R_3}{R_C} \right) \left( V_{BE2} + 2 V_T \left( \frac{R_2}{R_C} \right) \ln(\alpha) \right) \Rightarrow K = \frac{2}{R_2} \ln(\alpha)
\]
EXAMPLE!

DESIGN A 2.5V BANDGAP REFERENCE WITH $T_{CF} = \phi @ 300^\circ K$

1st, FIND REQUIRED $V_0$ (BEFORE SCALING)

$$V_0 = V_{60} + V_t (\gamma - \alpha) (1 + \ln \left( \frac{V}{T} \right))$$

$$V_0 = 1.205 + (25.86 \times 10^{-3}) \times 2.2 \quad @ 300^\circ K, \quad \alpha = 3.2, \quad \gamma = 1$$

$$V_0 = 1.2619 \text{V}$$

$$. \quad V_{out} = (1.2619) \left( 1 + \frac{R_5}{R_6} \right) = 2.5 \quad \Rightarrow \quad \frac{R_5}{R_6} = 0.98$$

$$. \quad \text{USE:} \quad R_5 = 9.8 \text{K}, \quad R_6 = 10 \text{K}$$

NOW,

$$1.2619 = V_{BE2} + 2 \frac{V_t}{R_6} \ln (2) \left( \frac{R_5}{R_6} \right)$$

ALSO,

$$I_2 = I_1 = \Delta V_{BE} \left( \frac{R_t}{R_2} \right)$$

$$\Rightarrow \quad 1.2619 = \frac{V_t}{R_6} \ln \left( \frac{R_5}{R_6} \right) + 2I_2 R_1$$

ASSUME: $I_{st} = 10^{-15}$A, USE: $R_1 = 1 \text{K}$, & ITERATE TO SOLVE

$$\Rightarrow I_2 = 289.7 \mu\text{A} \quad \Rightarrow \quad R_2 = 61.9 \Omega$$

$$(\Rightarrow K = 2 \left( \frac{R_6}{R_5} \right) \ln (\alpha) = 22.405)$$

NOTE IMPORTANCE OF:

1) MATCHING BETWEEN: $R_1$ & $R_2$ (TO SET $R$)

$R_3$ & $R_4$ (TO SET $I$)

2) ABSOLUTE VALUE OF $R_2$ - TO SET $I_1, I_2$

AND $I_{BE2}$
When bias is distributed as a voltage, errors occur due to:

1. Device mismatches ($\Delta V_I$, $\Delta B$)
2. Differences in ground/VDD potentials (e.g., IR drops)
3. Noise pickup & crosstalk

This situation can be improved by distributing biases as currents:

![Diagram of current distribution](image)

**Figure 18.21** Distribution of a reference voltage for current-mirror biasing.

**Figure 18.22** Distribution of current to reduce the effect of interconnect resistance.

*Rule of Thumb:*

When distributing biases over distance (e.g., across chip to multiple blocks), use currents. Inside a single block, use voltages (e.g., inside an op amp) to save area.
BIAS FILTERS:

WHEN BIAS CURRENTS ARE ROUTED ACROSS CHIP (E.G., FROM A MASTER BIAS GENERATOR TO MULTIPLE BLOCKS) THEY OFTEN PICK UP NOISE FROM OTHER SIGNALS, SUBSTRATE COUPLING, ETC. TO CLEAN THIS UP, USE FILTERS AT THE RECEIVING BLOCK.

EXAMPLE:

```
\[ \text{\textbf{\textit{7}}} \text{\textbf{\textit{I}}}} \text{ NETWORK}
```

"\textit{PI}" NETWORKS ARE OFTEN USED TO LOWPASS FILTER INCOMING BIASSES TO REMOVE NOISE. NOTE THAT!

1. THE POLE FREQUENCY OF THE FILTER MUST BE CAREFULLY CHOSEN TO FILTER OUT EXPECTED NOISE SOURCES, SUCH AS CLOCKS (BUT DON'T WASTE AREA BY SETTING IT TOO LOW!)

2. THE MAIN FILTERING IS PROVIDED BY \( R C_2 \), SO \( C_0 >> C_1 \) IS TYPICAL.

3. \( C_1 \) IS ADDED TO PREVENT VGS OF \( M_B \) FROM VARYING TOO MUCH, WHICH CAN LEAD TO A SHIFT IN BIAS DUE TO NONLINEAR RECTIFICATION SINCE FETS ARE SQUARE-LAW DEVICES (I.E., THE NOISE CAN HAVE A DC COMPONENT)

4. \( C_1, C_L \) ARE TYPICALLY BUILT FROM MOS CAPS TO SAVE AREA.

5. WHEN WORKING IN DEEP SUB-MICRON PROCESSES (I.E., \( 0.12 \mu m \) AND BELOW), WATCH OUT FOR IR DROPS ACROSS THE RESISTOR DUE TO GATE LEAKAGE!
When distributing references across chip (e.g. from a central bandgap), these can pick up noise and errors due to IR drops similar to biases! ⇒ Solution = Distribute references as currents!

\[ I_1, I_2, \ldots \text{sent to circuits needing references} \]

**NOTES:**

1. \( V_{R\,(\text{out})} = \alpha \frac{V_{\text{ref}}(R)}{R} \) = Scaled version of original reference voltage, based on resistor ratios and current mirror gain \( \alpha \) only (process, temp variations cancel).

2. To improve matching between \( R_1 \) & \( R_2 \), use unit resistors (exact same layout!) and watch orientation.

3. Watch out for errors due to: opamp \( V_{os} \) and gain errors, current mirror mismatches, output impedance of current mirror.

4. Often buffer \( V_{R\,(\text{out})} \) with an opamp to provide enough current capability & low \( R_{out} \), plus add bypass cap.
**Absolute Current Bias**

It is often desirable to provide a current bias to circuits which is absolute accurate (e.g., current DACs). One way to do this is through the use of an external precision resistor:

![Circuit Diagram]

\[ I_{\text{ref}} = \frac{V_{\text{ref}}}{R_{\text{external}}} = \text{absolute accurate if } V_{\text{ref}} \text{ & } R_{\text{external}} \text{ are precision elements} \]

However, the capacitance encountered going off-chip can be quite large, causing the pole at the negative input of the op amp to be relatively low frequency => problems with compensation!

Two solutions exist for this:

1) **Make the pole at the - input of the op amp the dominant pole**

   **Drawback:** the op amp must not have any other low frequency poles, which greatly complicates its design

   **Advantage:** everything is inside the feedback loop, increasing accuracy

2) **Use "Replica Biasing"**

   **Drawback:** since the replica leg is outside the feedback loop, some accuracy is lost

   **Advantage:** much easier op amp design!
REPLICA BIASING

WHEN A LARGE CAPACITANCE IS REQUIRED IN A FEEDBACK LOOP (E.G., FOR FILTERING OR TO GO OFF CHIP), IT'S OFTEN BETTER TO USE A "REPLICA" OF THE BIAS LEG FOR THE FEEDBACK:

![Diagram of replica biasing](image)

HERE AN ON-CHIP "REPLICA" OF THE MAIN BIAS LEG IS USED TO CLOSE THE FEEDBACK LOOP, THUS KEEPING THE LARGE PARASITIC CAP, \( C_p \), ENCOUNTERED GOING OFF CHIP OUT OF THE FEEDBACK LOOP

ADVANTAGE! WITH \( C_p \) OUT OF THE FEEDBACK LOOP, THE OP AMP IS MUCH EASIER TO DESIGN AND COMPENSATE

DISADVANTAGE: SINCE THE REFERENCE LEG USED TO CREATE \( I_{REF} \) IS OUTSIDE THE FEEDBACK LOOP, SOME ACCURACY IS LOST (I.E., IS THE VOLTAGE ON THE PAD = \( V_{REF} \)?)

LOOK OUT FOR:

1) MATCHING BETWEEN \( M_N \) & \( M_{NR} \)

2) ROUT LOOKING INTO SOURCE OF \( M_N \)
   (NEEDS TO BE LOW, TO REDUCE ERRORS DUE TO \( I_{REF} \neq I_A \))