Supply Independent Biasing

- **NEED SUPPLY-DEPENDENT BIASING TO**:
  1) **PROVIDE STABLE OPERATING POINTS**
     - **DEVICES**: $E_f$, $g_m$, $v_o$, $f_t$, etc.
     - **AMPS**: $A_v$, $B_z$, $S_R$, etc.
  2) **MINIMIZE POWER DISSIPATION**
     - **ALLOW MINIMUM BIAS CURRENTS FOR**
     - **A GIVEN APPLICATION**
  3) **REJECT POWER SUPPLY NOISE**

- **PROBLEM**: NEED TO REFERENCE BIAS CURRENTS
  TO SOMETHING BESIDES VDD!

- **POSSIBLE REFERENCES**:
  - $V_{BE}$
  - $V_{BE} = \frac{RI}{B}$ \((\Delta V_{BE} \text{ causes})\)
  - $V_{GBE}$
  - $V_T$ \((MOS \text{ thresholds})\)
  - $\Delta V_T$
  - $\Delta V_{GB}$
**Simple Bias Circuits**

The sensitivity of $I_{c2}$ to variations in $V_{cc}$ is:

$$S_{I_{c2}} = \frac{\% \text{ change in } I_{c2}}{\% \text{ change in } V_{cc}}$$

$$= \frac{\Delta I_{c2}}{I_{c2}} \cdot \frac{V_{cc}}{\Delta V_{cc}} = \frac{V_{cc}/I_{c2}}{\Delta I_{c2}/\Delta V_{cc}} = \frac{V_{cc}}{I_{c2}} \cdot \frac{2I_{c2}}{2V_{cc}}$$

$$= (R) \frac{2V_{cc}}{2V_{cc}} = R \left( \frac{1}{R} \right) = 1$$

$$I_{c2} \approx 1 \Rightarrow \text{ any } \Delta V_{cc} \text{ is transmitted through } R \text{ to } \Delta I_{c2}!$$

**Diode Source**

$$I_{c2} \cdot R_2 = V_t \cdot \ln \left( \frac{I_{ref}}{I_{c2}} \right)$$

**Algebra**

$$\frac{2I_{c2}}{V_{cc}} = \left( \frac{1}{1 + I_{c2} R_2} \right) I_{c2} \cdot \frac{2I_{ref}}{V_{cc}}$$

$$\Rightarrow S_{I_{c2}} = \frac{\left( \frac{1}{V_t} \right) I_{ref}}{\Delta V_{cc}} \approx 1$$

Typical $\# = .15 - .20$

$10\% \Delta V_{cc}$ would cause a $1.5 - 2\% \Delta I_{c2}$

$\Rightarrow$ Lower dependence of $I_{c2}$ on $V_{cc}$ is due to log dependence of $V_{be}$ on $I_{ref}$

*Often this is still not good enough.*
VBE-REFERENCED BIAS Ckt (Bipolar Process)

Figure 4.24a Supply-independent bias using \( V_{be} \) as a reference.

Figure 4.24b Self-biasing \( V_{be} \) reference.

Figure 4.24c Determination of operating point.

\[
I_{out} = \frac{V_{be1}}{R_2} = \frac{V_t \ln(I_{ref})}{R_2} \\
I_{ref} \approx \frac{V_{cc} - 2V_{be}}{R_1}
\]

**Better, but \( I_{ref} \) still is \( \approx V_{cc} \)**

For Ckt (b) \( \Rightarrow \)

\[
I_{out} = I_{c2} = I_{ref} \Rightarrow \\
I_{out} = \frac{V_{be1}}{R_2} = \frac{V_t \ln(I_{out})}{R_2}
\]

*Here, \( I_{out} \) is independent of \( V_{cc} \)!
(except for \( V_t \) effects)*

**Also, need start-up circuit!**

Figure 4.24d Start-up circuitry to avoid zero-current state.
\[ TCF = \text{"Fractional Temperature Coefficient"} = \frac{1}{I_{out}} \frac{2I_{out}}{\partial I}\frac{\partial I}{\partial T} \]

For the \( V_{BE} \)-referenced circuit of Fig. 4.24b,

\[
I_{out} = \frac{V_{BE}}{R} \tag{4.166}
\]

\[
\frac{\partial I_{out}}{\partial T} = \frac{1}{R} \frac{\partial V_{BE}}{\partial T} - \frac{V_{BE}}{R^2} \frac{\partial R}{\partial T} \tag{4.167}
\]

\[
= I_{out} \left( \frac{1}{V_{BE}} \frac{\partial V_{BE}}{\partial T} - \frac{1}{R} \frac{\partial R}{\partial T} \right) \tag{4.168}
\]

Therefore,

\[
TC_F = \frac{1}{V_{BE}} \frac{\partial V_{BE}}{\partial T} - \frac{1}{R} \frac{\partial R}{\partial T} \tag{4.169}
\]

Thus the temperature dependence of \( I_{out} \) is related to the difference between the resistor temperature coefficient and that of the base-emitter junction. Since the former has a positive and the latter a negative coefficient, the net \( TC_F \) is quite large.

**Example**

Design a bias reference as shown in Fig. 4.24b to produce 100 \( \mu \)A output current. Find the \( TC_F \). Assume that, for \( Q_1 \), \( I_C = 1 \times 10^{-14} \) A. Assume that \( \partial V_{BE}/\partial T = -2 \) mV/°C and that \((1/R) \partial R/\partial T = -1500 \) ppm/°C.

The current in \( Q_1 \) will be equal to \( I_{out} \), so that

\[ V_{BE} = V_{BE} \times \frac{100 \mu A}{10^{-14} \text{ A}} = 598 \text{ mV} \]

Thus, from (4.166),

\[ R = \frac{598 \text{ mV}}{0.1 \text{ mA}} = 5.98 \text{ k}\Omega \]

From (4.169),

\[ TC_F = \frac{-2 \text{ mV/°C}}{598 \text{ mV}} - 1.5 \times 10^{-3} = -3.3 \times 10^{-3} - 1.5 \times 10^{-3} \]

and thus

\[ TC_F = -4.8 \times 10^{-3}/\text{°C} = -4.8 \text{ ppm/°C} \]

The term ppm is an abbreviation for parts per million, and implies a multiplier of \( 10^{-6} \).

**Note:** Since \( TC_F \) is negative for \( V_{BE} \) & positive for \( R \), these add! \( \Rightarrow \text{large } TC_F \text{ for this circuit!} \)
$V_{+} - $ REFERENCED BIAS Ckt. ($\Delta V_{BB}$) (Bitolux Process)

Figure 4.19  (a) Bias source utilizing the thermal voltage. (b) Determination of operating point.

Q1,2 $\Rightarrow$ $V_{BE1} = V_{BE2} + I_R \frac{R_2}{R_{2}}$, WHERE: $I_2 = I_{C2} = I$,

$$I = \frac{V_{BE1} - V_{BE2}}{R_2} \quad \text{DUE TO Q3,4 MIRROR}$$

NEXCT BASE CURRENTS

NOW, $\Delta V_{BB} = V_{BB1} - V_{BB2} = V_{BE1} \ln\left(\frac{I_{S1}}{I_{S2}}\right) - V_{BE2} \ln\left(\frac{I_{S2}}{I_{S1}}\right)$

$$\Delta V_{BB} = V_{BE} \ln\left(\frac{I_{S2}}{I_{S1}}\right)$$

IF: $I_{S2} = K I_{S1}$ (Q2 HAS K TIMES THE EASIER AREA)

$$I = \frac{V_{BE} \ln(K)}{R_2} \quad \text{INDEPENDENT OF VCC!}$$

(EXCEPT FOR Vo EFFECTS)
The temperature variation of the output current can be calculated as follows.

From (4.172)

\[
\frac{1}{I_{C_1}} \frac{\partial I_{C_2}}{\partial T} = \frac{1}{I_{C_2}} \frac{\partial}{\partial T} \left( \frac{V_T}{R_2} \ln 2 \right) \tag{4.173}
\]

\[
= \frac{1}{I_{C_1}} \frac{V_T}{R_2} \left( \ln 2 \right) \left( \frac{1}{V_T} \frac{\partial R_1}{\partial T} - \frac{1}{R_2} \frac{\partial V_T}{\partial T} \right) \tag{4.174}
\]

and using (4.172) again we find

\[
TC_F = \frac{1}{I_{C_1}} \frac{\partial I_{C_2}}{\partial T} = \left( \frac{1}{V_T} \frac{\partial V_T}{\partial T} - \frac{1}{R_2} \frac{\partial V_T}{\partial T} \right) \tag{4.175}
\]

We have chosen a transistor area ratio of two to one as an example. Actually, this ratio is chosen to minimize the total area required for the transistors and for \( R_b \)

\[
I_{C_1} = \frac{V_T}{R_2} \ln 2 \quad \tag{4.172}
\]  

\(-\left( \text{For } k = 2 \right)\)

**EXAMPLE**

Design a bias circuit of the type shown in Fig. 4.13 to produce an output current of 100 \( \mu \)A. Find the \( TC_F \) of \( I_{out} \). Assume base-diffused resistors, \( 1/\beta (dR/dT) = -1500 \text{ ppm/°C} \).

From (4.172)

\[
I_{out} = \frac{V_T}{R_2} \ln 2
\]

which gives

\[
R_2 = \frac{126 \text{ mV} \ln 2}{100 \ \mu \text{A}} = 180 \ \Omega
\]

From (4.175)

\[
TC_F = \frac{1}{I_{out}} \frac{\partial I_{out}}{\partial T} = \frac{1}{V_T} \frac{\partial}{\partial T} (V_T) - 1500 \times 10^{-6} = \frac{1}{V_T} \left( \frac{V_T}{T} \right) - 1500 \times 10^{-6}
\]

\[
= \frac{1}{T} - 1500 \times 10^{-6}
\]

Assuming room temperature, \( T = 300 \text{°K} \),

\[
\frac{1}{I_{out}} \frac{\partial I_{out}}{\partial T} = 3000 \times 10^{-6} - 1500 \times 10^{-6} = 1800 \text{ ppm/°C}.
\]

**NOTE:** The smaller \( TC_F \), since the temperature coefficients of both \( V_T, R_2 \) are positive and therefore tend to cancel (but not completely).
VBE & ΔVBE (Vt) Referenced Bias Cuts in CMOS

**WITH M1, M2 MATCHED**

\[
I_{R} = V_{BE1} = V_{T} \ln \left( \frac{I_{S}}{I_{T}} \right)
\]

OR

\[
I = \frac{V_{BE} \ln (\frac{I_{S}}{I_{T}})}{R}
\]

(SAME AS FOR THE BIPOLAR PROCESS CASE)

**FOR THIS CIRCUIT,**

\[
I_{R} = \Delta V_{BE},
\]

\[
\Rightarrow I = \frac{V_{T} \ln (m)}{R}
\]

(AS BEFORE FOR THE BIPOLAR PROCESS CASE)

**NOTE:** THESE CIRCUITS HAVE THE ADVANTAGE THAT VBE IS WELL CONTROLLED, HOWEVER, ΔV IN VT (THRESHOLD) BETWEEN M1 & 2 HAVE A LARGE EFFECT!

THE ABOVE CIRCUITS MAKE USE OF THE SUBSTRATE PNP AVAILABLE IN AN N-WELL CMOS PROCESS;

IN A T-WELL PROCESS THE SUBSTRATE NPN COULD BE USED IN A SIMILAR FASHION.
Figure 12.28 Example of a 1/2-referenced self-biased reference circuit with cascoded devices to improve power-supply rejection and initial accuracy.

1) CASCODES USED TO AVOID (REDUCE) ERRORS DUE TO ΔVDS ACROSS DEVICES (V0 OR 2 ERRORS)

2) LARGE GEOMETRY, WELL-MATCHED DEVICES TYPICALLY USED FOR M1, M2.
Figure 5.5-7  (a) Breakdown diode voltage reference. (b) Small-signal model of Zener diode.

\[ \Delta I_d = \frac{\Delta V_{BE}}{\Delta V_{BE}} \left( \frac{V_{BE}}{V_{BE}} \right) \approx \left( \frac{V_{BE}}{V_{BE}} \right) = \left( \frac{I_z}{V_z + R} \right) \frac{V_{BE}}{BV} \]  

**PROBLEMS:**
- **TYPICAL BREAKDOWNS (CMOS PROCESS) \( \approx 5 \text{-} 6 \text{ V} \)
- **TOF CAN BE HIGH**
- **NOISE**!

Figure 5.5-6  Variance of the temperature coefficient of the breakdown diode as a function of the breakdown voltage BV [5]. (By permission from John Wiley & Sons, Inc.)
VT & ΔVT REFERENCED CIRCUITS

**NOTE:** M1 (ABOVE) IS A DEPLETION-MODE MOSFET, SO VT1 < 0

**PROBLEM:** THE ACCURACY OF THESE CIRCUITS DEPEND ON THE VALUE OF VT (THRESHOLD), WHICH CAN VARY WIDELY WITH PROCESS VARIATIONS!

**SIMILAR TO THE VBE GET!**

\[
I_R = \frac{V_{GS1}}{R} \Rightarrow I = \frac{V_{GS1}}{R}
\]

\[
= \left(\frac{1}{R}\right) \left[ V_T + \frac{2I}{\sqrt{\frac{2I}{(V_{GS}-V_T)}^2}} \right]
\]

IF \((V_{GS}-V_T) \gg V_T) \Rightarrow I \approx \frac{V_T}{R}

(IF NOT, USE EXACT EQ. ABOVE)

**FOR THIS CIRCUIT,**

\[
V_0 = V_{GS2} - V_{GS1}
\]

**SINCE**

\[
V_{GS} = V_T + \frac{2I}{\sqrt{(2I)^2}}
\]

\[
\Rightarrow I_{D1} = I_{D2}, \quad V_{\frac{1}{2}} = \frac{V_{\frac{1}{2}}}{2}
\]

\[
I_{D1} = I_{D2}
\]

\[
\Rightarrow V_0 = V_{GS2} - V_{GS1} = \Delta V_{GS}
\]

\[
V_0 = V_{D2} - V_{T1}
\]
\[ \Delta V_{ES} \text{ Bias Ckt} \]

\[ I_{out} = I_{p1} = I_{p2} = I \]

\[ I_{R} = V_{ES1} - V_{ES2} = \Delta V_{ES} \]

\[ \Rightarrow I = \frac{\Delta V_{ES}}{R} \]

Now, \( I_{p} = \frac{K}{2} \left( V_{ES} - V_{T} \right)^{2} \) \( \Rightarrow V_{ES} = V_{T} + \frac{2I}{K} \)

\[ I_{R} = \Delta V_{ES} = \sqrt{\frac{2I}{K(V_{T})}} - \sqrt{\frac{2I}{K(V_{T})}} \]

\[ \left( IR \right)^{2} = \left( \frac{2I}{K(V_{T})} \right) - 2 \left( \frac{2I}{K(V_{T})} \right) \left( \frac{2I}{K(V_{T})} \right) + \left( \frac{2I}{K(V_{T})} \right) \]

\[ I_{R}^{2} = \left( \frac{2I}{K(V_{T})} \right) \left[ \left( \frac{1}{2} \right)_{1} - 2 \left( \frac{1}{2} \right)_{1} + \left( \frac{1}{2} \right)_{2} \right] \]

\[ I = \left( \frac{2I}{K(V_{T})} \right) \left[ \left( \frac{1}{2} \right)_{1} - 2 \left( \frac{1}{2} \right)_{1} + \left( \frac{1}{2} \right)_{2} \right] \]

Let: \( \left( \frac{1}{2} \right)_{1} = K \left( \frac{1}{2} \right), \Rightarrow \left( \frac{1}{2} \right)_{1} = \left( \frac{1}{2} \right)_{2} \)

\[ I = \left( \frac{2I}{K(V_{T})} \right) \left[ 1 - 2 \sqrt{1 - \frac{1}{K}} \right] \]

Example: \( K = 50 \, \text{mA} \), \( \left( \frac{1}{2} \right) = \frac{100}{10} \), \( K = 4 \), \( I = 100 \, \text{mA} \) \( \Rightarrow R = 10 \, \Omega \)

Note: If this Ckt is used to bias an OTIMP, \( gm = \sqrt{2} \frac{K}{I_{p}} \), since \( I_{p} = \frac{V_{T}}{K} \) * independence of temperature or process variation (except for \( \Delta R \) i.e., \( I_{R} \) !) (however, \( gm- \gamma_{0} \) ) \( \gamma_{0} = \frac{1}{2} T_{m} \)
As seen previously,

$$T_C \approx -2 \text{ mV/}^\circ\text{C} \text{ for a PN diode}$$

and,

$$T_C \text{ varies from } \approx \phi \text{ to } 4 \text{ mV/}^\circ\text{C for a Zener diode} \text{ as } V_Z = 5-8 \text{ V}$$

since these $T_C'$s are of opposite sign, diodes can be used to compensate for the Zener $T_C$ to achieve overall $T_C \approx \phi$

In circuits (a) & (b) above,

$$V_{R2} \approx V_2 \Rightarrow I_{OUT} \approx \frac{V_2}{R_2}$$

which yields a zero $T_C$ for $I_{OUT}$ if the $T_C'$s of the Zener and resistor match. If not, additional diodes can be added in series with the Zener or the resistor, as needed (circuit (c)).
V_2 - (m+2)V_{BE(on)} = I_{OUT} R_2  \quad \text{WHERE} \quad m = \# \text{ DIODES}

\Rightarrow \quad \frac{\partial V_2}{\partial T} - (m+2) \frac{\partial V_{BE(on)}}{\partial T} = R_2 \frac{\partial I_{OUT}}{\partial T} + I_{OUT} \frac{\partial R_2}{\partial T}

\left[ \frac{1}{R_2 I_{OUT}} \right] \left[ \frac{\partial V_2}{\partial T} - (m+2) \frac{\partial V_{BE(on)}}{\partial T} \right] = \frac{1}{I_{OUT}} \frac{\partial I_{OUT}}{\partial T} + \frac{1}{R_2} \frac{\partial R_2}{\partial T}

OR,

\frac{1}{I_{OUT}} \frac{\partial I_{OUT}}{\partial T} = \left( \frac{1}{R_2 I_{OUT}} \right) \left[ \frac{\partial V_2}{\partial T} - (m+2) \frac{\partial V_{BE(on)}}{\partial T} \right] - \frac{1}{R_2} \frac{\partial R_2}{\partial T}

\text{FOR: } TCF\left(I_{OUT}\right) = \frac{1}{I_{OUT}} \frac{\partial I_{OUT}}{\partial T} = \phi

\Rightarrow

\left( \frac{1}{R_2 I_{OUT}} \right) \left[ \frac{\partial V_2}{\partial T} - (m+2) \frac{\partial V_{BE(on)}}{\partial T} \right] - \frac{1}{R_2} \frac{\partial R_2}{\partial T} = \phi

OR,

\left( \frac{1}{R_2 I_{OUT}} \right) \left[ \frac{\partial V_2}{\partial T} - (m+2) \frac{\partial V_{BE(on)}}{\partial T} \right] = \frac{1}{R_2} \frac{\partial R_2}{\partial T}

\text{SINCE THE LEFT SIDE OF THIS EQUATION = THE TCF OF THE VOLTAGE ACROSS } R_2 \text{, THIS BASICALLY SAYS THAT, FOR CONSTANT } I_{OUT} \text{, THE VOLTAGE ACROSS } R_2 \text{ MUST INCREASE AT THE SAME RATE AS } R_2! \text{.}

\Rightarrow \quad I_{OUT} = \frac{V_{RS}}{R_2} = \text{A CONSTANT!}
EXAMPLE

Determine the required values of \( n \) and \( R_1 \) in Fig. 4.26c to produce an output current of 100 \( \mu \)A with the lowest possible TC. Assume

\[
\frac{\partial V}{\partial T} = +2.5 \text{ mV/}^\circ\text{C}; \quad V_2 = 6.1 \text{ V}
\]

\[
\frac{1}{R} \frac{\partial R}{\partial T} = -2 \text{ kV/mV/}^\circ\text{C}
\]

\[
\frac{\partial V_{\text{Re}}}{\partial T} = -2 \text{ mV/}^\circ\text{C}; \quad V_{\text{Re}} = 0.6 \text{ V}
\]

From (4.180) we have

\[
0 = 2.5 \text{ mV/}^\circ\text{C} - (n + 2)(-2 \text{ mV/}^\circ\text{C}) - R_1 (I_{\text{sat}}/2000 \times 10^{-6})
\]

We also have the auxiliary condition from (4.177)

\[
V_2 = I_{\text{sat}}R_1 + (n + 2)V_{\text{Re}}
\]

Combining these relations,

\[
0 = 2.5 \text{ mV/}^\circ\text{C} - (n + 2)(-2 \text{ mV/}^\circ\text{C}) - [V_2 - (n + 2)V_{\text{Re}}] (2000 \times 10^{-6})
\]

Solving this equation,

\[
(n + 2) = 3.09
\]

\[
\eta = 1.09
\]

Utilizing (4.177) \( R_1 = 43.4 \text{ k} \Omega \).

\[
\text{SINCE } n \text{ IS NOT AN INTEGER (THE USUAL CASE), A VBE MULTIPLIER WAS USED TO SET } n = 1.09
\]

\[
\text{NOTE: THE DIODES WERE USED IN SERIES WITH THE RESISTOR, SINCE THE TC OF THE RESISTOR WAS HIGHER THAN THAT OF THE ZENER IN THIS CASE. IF THIS WERE REVERSED, THEY WOULD BE USED IN SERIES WITH THE ZENER!}
\]

\[
(2.5 \text{ mV/}^\circ\text{C}) \times 6.2 \text{ V} = 400 \text{ ppm}
\]