1. (30 points) An NMOS FET is biased in saturation with $V_{DS} = V_{DS\text{-sat}}$. If $V_{DS}$ is increased by 1V, what percent increase would you expect to see in the drain current? Use: $W/L = 20:18$ and for the drain-to-substrate PN junction use $N_A = 10^{19}/cm^2$, $N_D = 10^{19}/cm^2$, $\phi_B = 1V$ (Hint: think about what causes channel-length modulation!)

**Case 1:** $V_{DS} = V_{DS\text{-sat}} \Rightarrow V_D$ across PN junction = 0

From class notes:

\[ X_T = \left( \frac{2L_S}{D} \right) \left( \frac{V_D - V_P}{N_A} \right) \left( \frac{V_D - V_P}{N_D} \right) \]

where $D$ is the depletion region extends into the P side of junction.

OR, $X_D = \left( \frac{2L_S}{D} \right) \left( \frac{V_D - V_P}{N_A} \right)$ for $N_D >> N_A$, as in this case.

\[ X_D = \left[ \frac{2(1.17)(9.85 \times 10^{14})}{(1.6 \times 10^{-10})} \right]^{1/2} = 3.6 \times 10^{-6} \text{ cm} = 0.036 \mu\text{m} = X_P \]

**Case 2:** $V_{DS} = V_{DS\text{-sat}} + 1V \Rightarrow V_D$ across PN junction = -1

\[ X_T = \left[ \frac{2(1.17)(9.85 \times 10^{14})}{(1.6 \times 10^{-10})} \right]^{1/2} = \sqrt{2} \text{ (previous result)} = 0.051 \mu\text{m} = X_T \]

Now, in saturation $I_D = \frac{1}{2} L_1 \left( \frac{V_D}{L} \right) (V_{DS} - V_T)^2$ without $x$ term.

\[ \frac{I_D}{I_D} = \frac{1}{2} L_1 \left( \frac{V_D}{L} \right) (V_{DS} - V_T)^2 = \left( \frac{L_1}{L} \right) \left( \frac{V_D}{L} \right) (V_{DS} - V_T)^2 = \left( \frac{L_1}{L} \right) \left( \frac{V}{L} \right) (0.18 - 0.036) = 0.144 \]

\[ L_1 = 0.18 - 0.036 = 0.144 \]

\[ L_1 = 1.146 \]

\[ \Rightarrow I_D = (1.146)I_D \Rightarrow \text{IP increases by 11.6% as } \]

\[ V_{DS\text{-sat}} + 1V \]
2. (30 points) Find the voltage gain for the double cascode circuit shown at right. Assume $V_{bias1}$ and $V_{bias2}$ are chosen to keep all MOSFETs saturated and the bipolar in the forward-active region. Also assume the current source is ideal, and use the following values: $I_{bias} = 1mA, V_{B} = 25mV, \beta = 100, V_{A} = 100V, k'_{N} = 100\mu V^2/V^3, \lambda_{N} = 0.1, \gamma = 0$ (neglect body effect), $W/L_{3} = W/L_{4} = 20/4$

**1st, FIND VALUES FOR SMALL-SIGNAL PARAMETERS AT THE GIVEN BIAS POINT!**

**BJT:** \[ \frac{g_m}{V_T} = \frac{I_C}{25mV} = \frac{1mA}{25mV} = 40mV = g_m(BJT) \]
\[ Y_O = \frac{V_A}{I_C} = \frac{100 V}{1mA} = 100k\Omega = Y_0(BJT) \]

**FETs:** \[ g_m = \frac{1}{2R_{le}} = \left[ \frac{2(100\times10^6)(\frac{2}{\mu_A})(1\times10^3)}{0.1(1\times10^3)} \right] \frac{1}{1mA} = g_m(FET) \]
\[ Y_{sa} = \frac{1}{2R_s} = \frac{1}{0.1(1\times10^3)} = 10k\Omega = Y_{sa}(FET) \]

**Now, FIND $R_{out}$ = RESISTANCE LOOKING INTO BASE OF M3 (SINCE IDEAL CURRENT SOURCE HAS $0 R_{out}$)**

\[ R_{out} = R_{o3} = Y_{sa3} (1 + g_m3 R_{o2}) \]
\[ R_{o2} = Y_{sa2} (1 + g_m2 R_C) \]
\[ R_{o2} = (10k\Omega)(1 + (1mA)(1.01k\Omega)) = (10k\Omega)(101) = 1.01m\Omega \approx 1m\Omega = R_{o2} \]
\[ R_{o3} = (10k\Omega)(1 + (1mA)(1.01k\Omega)) = (10k\Omega)(1001) = 10.01m\Omega \approx 10m\Omega = R_{o3} \]

**THEN,** \[ A_V = \frac{g_m}{R_{out}} = \frac{40mV}{(10m\Omega)(10m\Omega)} = \frac{-400,000}{11228} \]

**ASSUME:** Note that $M_F(BJT) = g_mY_0 = 4000$, $M_F(FET) = g_mY_{sa} = 10$

**HERE, SINCE LOAD = IDEAL CURRENT SOURCE,**
\[ A_V = (M_F(BJT))(M_F(MOS))(M_F(1000)) = (4000)(10)(10) = 400,000 \]

**DUE TO DOUBLE-CASCODE!**
3. (20 points each) For each of the amplifiers shown below, find the differential voltage gain.
For amp A assume the DC bias voltage across $R_1$ and $R_2$ is 2V, and $r_o >> R_1$ and $R_2$.
For amp B use $V_{A_{NNN}} = 150V$, $V_{A_{PP}} = 30V$. For both A and B use: $V_i = 25mV$, $r_o = 100$

**Differential amp A**

**A. FOR AMP A, $A_V = \frac{1}{2} \frac{g_m \cdot R_L}{V_{RL}}$ (1/2 DUE TO SINGLE-ENDED OUTPUT)**

$$A_V = \frac{1}{2} \left( \frac{V_{RL}}{V_L} \right) = \frac{1}{2} \left( \frac{V_{RL}}{V_L} \right)$$

**WHERE: $V_{RL} =$ DC BIAS VOLTAGE ACROSS $R_1, R_2$**

$$\Rightarrow A_V = \frac{1}{2} \left( \frac{2V}{25mV} \right) = \frac{25V}{25mV} = 100 = A_V$$

**NOTE: THIS ASSUMES $V_o >> R_1, R_2$ AS WAS GIVEN**

**B. FOR AMP B, $A_V = g_m \left( \frac{I_{ON}}{I_{OFF}} \right)$ (NO FACTOR OF 1/2 DUE TO CURRENT MIRROR)**

**NOW, $Y_o = \frac{V_A}{I_C}$**

$$\Rightarrow Y_o \left( \frac{I_{ON}}{I_{OFF}} \right) = \left( \frac{V_A}{I_C} \right) \left( \frac{I_{ON}}{I_{OFF}} \right) + \left( \frac{V_A}{I_C} \right)$$

$$\Rightarrow Y_o \left( \frac{I_{ON}}{I_{OFF}} \right) = \frac{1}{I_C} \left[ \left( \frac{V_A}{I_C} \right) \left( \frac{1}{I_{OFF}} \right) \right] = \frac{1}{I_C} \left[ \frac{(150)(30)}{(150 + 30)} \right]$$

$$= \frac{25V}{I_C} \Rightarrow 25V = A_V "EFFECTIVE" FOR NPN PNP$$

$$\Rightarrow A_V = g_m \cdot R_O \Rightarrow \left( \frac{25V}{I_C} \right) \left( \frac{V_{A_{NNN}}}{I_C} \right) = \frac{25V}{25mV} = 1000$$

$$\Rightarrow A_V = 1000$$
BONUS (5 points): Using similar device sizes and bias conditions, would you expect the gain for the amp shown below to be larger or smaller than the gain of the amp in problem 2? Explain your answer!

THE GAIN FOR THIS AMP WOULD BE SMALLER THAN THE AMP IN PROBLEM 2, FOR 2 REASONS:

1) \( A_v = g_m R_{out} \), WHERE \( g_m \) = TRANSCONDUCTANCE OF THE INPUT DEVICE

Since \( g_m \) (BJT) > \( g_m \) (FET), the amp in problem 2 has a higher \( g_m \) for the input device.

2) DOUBLE-CASCODING BJT's (AS THIS AMP Does), IS NOT EFFECTIVE SINCE \( R_{L} \) LIMITS AT \( \infty \) \( V_o \) (1+\( \beta \))

NOTE THAT NO SUCH LIMIT EXISTS FOR MOSFETS! SO, DOUBLE-CASCODING FETS (AS IN PROBLEM 2) DOES RAISE \( R_{out} \)!