MOS REVIEW

Fig. 1.7 Commonly used symbols for n-channel transistors.

Fig. 1.8 Commonly used symbols for p-channel transistors.

PREFERRED SYMBOLS:

SUBSTRATE ASSUMED CONNECTED TO APPROPRIATE SUPPLY (NMOS = VSS, PMOS = VDD)

IF EXPLICIT SUBSTRATE CONNECTION IS NEEDED (E.G., BULK TIED TO SOURCE)

PMOS IS SIMILAR, BUT IN N-WELL
• Designer's control W and L \((W = \text{key!})\)

• \(K_{ox}\), dimensions for \(S, D\) typically fixed for a given process (HPF)

**BASIC OPERATION**

![Diagram of MOS transistor operation](image)

- **Accumulation** (rarely used)
- **Triode region**

**PMOS is very similar but with all voltages reversed**

\(V_{GSS} < 0\) to form channel
Now, a MOSFET "turns on" when \( V_{GS} \) is large enough for a channel to form under the gate.

\[ V_T = \text{"Threshold Voltage"} = \frac{V_{GS}}{V_{GS}} \text{ required for the surface under the gate to "invert"} \]

- The amount of charge under the gate depends on how much \( V_{GS} \) exceeds \( V_T \):

\[ V_{EFFECTIVE} = \text{"Effective Gate-Source Voltage" (=} V_{ON}) \]

\[ V_{EFFECTIVE} = V_{GS} - V_T \]

\[ Q = CV = C_{OX} (V_{GS} - V_T) = \text{charge per unit area} \]

\[ Q_{TOTAL} = (\text{Area}) C_{OX} (V_{GS} - V_T) \]

\[ Q_{TOTAL} = WL C_{OX} (V_{GS} - V_T) = WL C_{OX} V_{EFFECTIVE} \]

Where: \( C_{OX} = \frac{E_{OX}}{e_{OX}} \text{ capacitance per unit area} \)

For SiO2: \( C_{OX} = \frac{3.9 E_0}{e_{OX}} \]

\[ E_0 = 8.85 \times 10^{-12} \text{ F/m} \]

\[ \epsilon_{OX} = \text{permittivity of free space} \]

\[ \Rightarrow C_{GS} = WL C_{OX} = \text{total gate capacitance} \]

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*NOTE: Nature rarely works like a light switch - some current does flow for \( V_{GS} < V_T \), "sub-threshold" operation (discussed later)*
**Regions of Operation:**

**Cutoff:** \( V_{GS} < V_T \) (or sub-threshold)

**Triode:** \( V_{GS} > V_T, \ V_{DS} < V_{DS-sat} \)

**Saturation:** \( V_{GS} > V_T, \ V_{DS} \geq V_{DS-sat} \)

\[ I_D = \frac{V_{DS} - V_{In}}{2L(V_{DS} - V_{In})} \]

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TRIODE REGION

\[ I_D = \mu \frac{W}{L} \left[ (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right] \quad (V_{DS} < V_{GS} - V_T) \]

- **CHANNEL EXTENDS ALL THE WAY FROM SOURCE TO DRAIN**

- **MOST COMMON USE OF MOSFET IN TRIODE REGION IN ANALOG CIRCUITS IS FOR A SWITCH (E.G., SWINDED-CAPACITOR CIRCUITS)**

7. \( V_G - V_{GS}(x) \) is the gate-to-channel voltage drop at distance \( x \) from the source end, with \( V_G \) being the same everywhere in the gate, since the gate material is highly conductive.
When \( V_{OS} \) is increased so that \( V_{OS} < V_{TH} \), the channel becomes pinched off at the drain end.

8. Because of the body effect, the threshold voltage at the drain end of the transistor is increased, resulting
in the true value of \( V_{TH-actual} \) being slightly lower than \( V_{TH} \).
9. Historically, the active region was called the saturation region, but this led to confusion because in the
case of bipolar transistors, the saturation region occurs for small \( V_{GS} \), whereas for MOS transistors it
occurs for large \( V_{GS} \). The renaming of the saturation region to the active region is beginning widely
accepted.

\[ \Delta L = \sqrt{V_{DS} - V_{TH} + \Phi_B} \]

- **When** \( V_{GATE} - V_{CHANNEL} (x) < V_T \) the channel becomes "pinched-off".
- **When this occurs**, the part of \( V_{PS} > V_{GS} - V_T \) appears across the depletion region between the
end of the channel and the drain.

\[ V_{DRAIN} = V_{GS} - V_T \]

Substituting \( V_{DS} = V_{GS} - V_T \) into our previous equation for \( I_D \) in triode yields:

\[ I_D = \frac{m}{2} C_{ox} \left( \frac{V_T}{2} \right) \left( (V_{GS} - V_T) (V_{GS} - V_T) - \frac{(V_{GS} - V_T)^2}{2} \right) \]

or

\[ I_D = \frac{m}{2} C_{ox} \left( \frac{V_T}{2} \right) (V_{GS} - V_T)^2 \quad \text{for} \quad V_{DS} \geq V_{GS} - V_T \]
**Channel Length Modulation**

- As discussed, in saturation the channel is "pinched off" near the drain.
- Any $V_{GS} > V_{GS,sat}$ appears across the depletion region between the channel and the drain.
  - As $V_{GS}$ changes, so does the width of this depletion region!
  - Therefore, $L_{effective} = L_{drawn} - X_{D}$ (Depletion Region Width)
  - $L_{effective}$ gets shorter and $I_{D}$ increases as $V_{GS}$ increases.
  - "Channel Length Modulation"!

![Graph showing $I_D$ vs $V_{GS}$ for different values of $V_D$.]

- New equation for $I_D$ in saturation:
  \[ I_D = \frac{m}{2} \cdot Cox \left( \frac{W}{L} \right) \left( V_{GS} - V_T \right)^2 \left( 1 + \frac{2}{V_{GS} - V_{GS,sat}} \right) \]

- Note that $\lambda = \frac{1}{\nu A} = \frac{1}{\text{Early Voltage}}$ (similar to BJT's)

- Simple equations for $\lambda$ are often wrong due to short-channel effects, etc., so $\lambda$ is typically measured for a given process.

- $\lambda$ is a strong function of $L_{drawn}$ ($\propto \frac{1}{L}$).
**Body Effect**

- So far we have assumed that $V_{threshold} = constant$ for a given FET, which is only true if $V_{SB} = 0$

- When the voltage from source-to-bulk ($V_{SB} \neq 0$), then $V_t$ increases because the depletion region from channel to substrate becomes wider, forcing more of the electric field lines from the gate to terminate on the immobile ions uncovered.

  \[ \Rightarrow \text{"Body Effect"!} \]

- Effects both large-signal & small-signal

  \[ V_T = V_{T0} + \Delta V_T \]
  \[ = V_{T0} + \gamma \left( \sqrt{V_{SB} + 12\phi_F} - \sqrt{12\phi_F} \right) \]

  Where:
  \[ \gamma = \sqrt{\frac{2qN_A k_S E_0}{C_{ox}}} \]
  \[ \phi_F = -\frac{kT}{q} \ln \left( \frac{N_A}{n_i} \right) = \text{Fermi potential of the substrate} \]

And:
  \[ q = \text{electron charge} = 1.602 \times 10^{-19} \text{ C} \]
  \[ N_A = \text{doping concentration} \]
  \[ k_S = \text{relative permittivity of Si} = 11.8 \]
  \[ E_0 = \text{permittivity of free space} \]
  \[ n_i = \text{intrinsic carrier concentration} \]
**Sub-Threshold Operation**

- So far, we have assumed that \( I_d = 0 \) for \( V_{GS} < V_T \) → NOT STRICTLY TRUE!

- In fact, for \( V_{GS} - V_T < \approx -100\text{mV} \) the FET is in weak inversion and operates in the "sub-threshold" region.

- In sub-threshold, the equations previously discussed for \( I_d \) do not apply; instead, \( I_d \) depends exponentially on \( V_{GS} \) (similar to a BJT):
  \[
  I_d = I_{d0} \left( \frac{W}{L} \right) e^{(V_{GS}/N_k T)}, \quad N \approx 1.5 - 2.0
  \]

- **Sub-threshold FETs Have**:
  - Higher \( g_m \), \( V_{th} \) → Higher Gain
  - Slower Speeds (Less \( I_d \) to Charge Cpts)

  \( \rightarrow \) Good for low power, slow applications (e.g., wristwatch)

**But ... THE BAD NEWS IS!**

- \( V_{GS} = 0 \) does NOT imply \( I_d = 0 \) → Leakage Current!

- How much leakage current you get depends on how far you are down the "sub-threshold slope".
OTHER 2ND ORDER EFFECTS:

1) P-N JUNCTION LEAKAGE CURRENTS (SOURCE, DRAIN)
2) GATE LEAKAGE (IMPORTANT FOR \( \approx 0.13 \mu m \) PROCESSES AND LOWER)
3) MOS PARASITIC CAPACITANCES (TO BE DISCUSSED LATER)

AND,

SHORT-CHANNEL EFFECTS:

1) MOBILITY DEGRADATION — E.G., VERTICAL E-FIELD AND MORE SURFACE COLLISIONS
2) VELOCITY SATURATION:
   \[
   \qquad \text{carrier} = V_d = \frac{V}{E} \quad \text{NO LONGER HOLDS!}
   \]
3) INCREASED CHANNEL LENGTH MODULATION
   (\( L \) IS NOW A BIGGER FRACTION OF \( L \))
4) "DIBL" = "DRAIN INDUCED BARRIER LOWERING"
   (\( V_{t} \) ↓ AS \( V_{ds} \) ↑ ⇒ \( R_{ds} \) ↓)
5) "HOT" CARRIER EFFECTS
   → DRAIN-TO-SUBSTRATE CURRENT DUE TO IMPACT IONIZATION AND AVALANCHEING (LIMITS \( R_{ds} \))
   → CARRIERS INJECTED INTO THE OXIDE (SHIFTS \( V_{t} \))
   → PUNCH THROUGH FROM DRAIN-TO-SOURCE
   (MORE TO COME AS PROCESSES GET SMALLER!)

WHAT IS A CIRCUIT DESIGNER TO DO, ??

⇒ SPICE MODELS IT ALL FOR YOU!
(BUT, YOU NEED GOOD MODELS!)
TO GET THE SMALL-SIGNAL PARAMETERS, TAKE DERIVATIVES OF THE LARGE SIGNAL EQUATIONS:

\[
g_m = \frac{\partial I_p}{\partial V_{gs}} = 2 \frac{\beta}{\varepsilon_0} \frac{(V_{gs}-V_T)^2}{(1+\alpha V_{ds})} \\
g_m \approx \beta (V_{gs}-V_T)
\]

OR, SUBSTITUTING INTO OUR OTHER EQUATIONS:

\[
g_m = \sqrt{2\beta I_p} = \frac{2I_p}{V_{gs}-V_T}
\]

AND, \( g_{de} = \frac{\partial I_p}{\partial V_{ds}} = 2 \left( \frac{\beta}{\varepsilon_0} \right) (V_{gs}-V_T)^2 \approx 2I_p \)

\[
\frac{1}{R_{de}} = g_{de} \approx 2I_p
\]
SMALL-SIGNAL MODELS (cont)

\[ A \text{mp}, \quad g_s = \frac{2I_E}{2V_{SB}} = \frac{2I_E}{2V_T} = \frac{2V_T}{2V_{SB}} = \ldots (\text{more}) \ldots \]

\[ g_{ds} = \frac{V_{gm}}{2 \sqrt{V_{SB} + 12I_E}} \quad "\text{back gate}" \]

Hmmm... given any set of large-signal equations, a small-signal model is easily derived.

To understand AC model, look at caps:

![Diagram of MOS transistor](image)

\[ C_{GS} \approx \frac{1}{3} C_{ox} W L \]

\[ C_{GD} \approx C_{overlap} = C_{ox} W L_{ov} \]

\[ C_{SB,PD} \text{ are } P-N \text{ junction caps} \]

\[ \rightarrow C_{j} = \frac{C_{PD}}{\sqrt{1 + \frac{V_{PD}}{V_{th}}} \ldots} \]
SMALL-SIGNAL MODELS FOR OTHER REGIONS

**TRIODE**

\[
\frac{1}{V_{th}} = \frac{g_{ds}}{2I_D} = \beta V_{GS}
\]

\[
C_{gs} = C_{gd} = \frac{1}{2} \text{Cox} W L
\]

**CUT-OFF**

\[
C_{gs} = C_{gd} = \text{Cox} W L_{DV}
\]

\[
C_{gb} = \text{Cox} W L (\text{max})
\]

REALLY DEPENDS ON DEPLETION REGION UNDER GATE (NO CHANNEL)
**Derivation of the Threshold Voltage**

First, examine the basic MOS system:

![Figure 6.2 Structure of the MOS system](image)

Now, if a gate voltage is applied, electric field lines extend through the oxide and down into the semiconductor, attracting charge to the surface (more on the charge later...)

![Figure 6.5 MOS electric fields](image)

This occurs because $F = qE$

Applies a force to push holes away from the surface and attract electrons towards the surface.
This electric field causes voltage drops.

![Diagram of MOS system with voltage drops](image)

**Figure 6.4** Voltages in the MOS system

- **The voltage drop across the oxide, \( V_{ox} \), can be found from the simple equation for a parallel-plate capacitor:**

  \[
  Q_s = C_{ox} V_{ox} \quad \Rightarrow \quad V_{ox} = \frac{Q_s}{C_{ox}}, \quad \text{where} \quad C_{ox} = \frac{E_{ox}}{\ell_{ox}}
  \]

  \( Q_s = \) charge in semiconductor

  and \( \phi_s = \) "surface potential"

- **Note that, since no electric field lines end on charge inside the gate oxide, the voltage drops linearly across the oxide according to:**

  \[ V = E \text{ (distance)} \]

- **However, since the electric field lines do terminate on charges in the silicon, the voltage drops slower and slower the deeper you go into the silicon (parabolic for a uniform charge density).**

Now, what does \( Q_s \) look like?
As we slowly increase the gate voltage, a depletion region forms in the silicon:

\[ V_G > 0 \text{ small} \]

![Diagram of MOS system with depletion region](image)

Figure 6.6 Bulk (depletion) charge in the MOS system

- Depletion region charge is due to fixed ions as the holes associated with acceptor dopant atoms are forced away from the surface.

→ Immobile charge, trapped in crystal lattice, cannot carry current

\[ Q_B = -\sqrt{2q\varepsilon e N_A \Phi_S} = \text{Bulk charge in depletion region} \]

Where: \( \varepsilon_S = 11.8 \varepsilon_0 = \text{Permittivity of Si} \)
\( q = 1.602 \times 10^{-19} = \text{Electron charge} \)
\( N_A = \text{Accepter dopant concentration} \)
\( \Phi_S = \text{Surface potential} \)
Now, as \( V_G \) is increased > \( V_T \) an inversion layer of electrons forms at the surface:

\[
\begin{align*}
+ V_G &> V_T \\
\text{Electron layer} &\rightarrow \text{Bulk charge} \\
Q_e &\rightarrow Q_B \\
Q_S &= Q_B + Q_e \\
\text{p-type} N_A &\quad \text{Bulk charge} \\
\end{align*}
\]

**Figure 6.7** Formation of the electron charge layer

*By definition, the inversion layer just starts to form when:

\[
Q_S = 2|\phi_F| = \text{Surface potential when inversion layer starts to form}
\]

Where: \( |\phi_F| = \frac{qE}{q} ln \left( \frac{N_A}{n_i} \right) = \text{Fermi potential in bulk} \)

**Note that:**

1. The inversion layer looks like a "sheet" of charge right at the silicon surface.

2. Once the inversion layer forms, further increases in \( V_G \) cause more charge in the inversion layer according to \( Q = C_G(V_G - V_T) \).

3. Once the inversion layer forms, the depletion region stops growing because the inversion layer "shields" it from new e-field lines from the gate (an approximation, but since the inversion layer grows exponentially with \( V_G \) and the depletion region is only growing \( \propto \sqrt{V_G} \), this is a very good approximation.)
Now, to calculate the threshold voltage, $V_T$, note that:

$$V_g = V_{ox} + \phi_s$$

So, when $V_g = V_t$:

$$V_e = V_T = V_{ox} + 2\phi_p$$  \(\text{since} \ \phi_s = 2\phi_p\)  

And since:

$$V_{ox} = \frac{q\varepsilon_s}{C_{ox}} = \frac{N_aq\varepsilon_s}{C_{ox}}$$

$$\Rightarrow V_T = \frac{1}{C_{ox}} \sqrt{2q\varepsilon_s N_a (2\phi_p)} + 2\phi_p$$  \(\text{ideal case}\)

Now, the above equation for $V_T$ neglects effects from non-idealities such as oxide charge and differences in material work functions, so add a term to account for this:

$$V_{FB} = \text{"Flatband Voltage"}$$

Also, most modern CMOS processes add an implant step to "Taylor" the $V_T$ values as desired:

$$\Rightarrow \Delta V_T = \frac{qD_i}{C_{ox}} = \text{shift in } V_T \text{ due to "threshold Tayloring implants"}$$

Thus the equation for $V_T$ becomes:

$$V_T = \frac{1}{C_{ox}} \sqrt{2q\varepsilon_s N_a (2\phi_p)} + 2\phi_p + V_{FB} + \frac{qD_i}{C_{ox}}$$

Where: $D_i$ = implant dose in atoms/cm$^2$

*Same for pMOS, just use $N_p$ instead of $N_a$

* Above eq assume $V_{SB} = \phi$ (no body effect)
EXAMPLE:

FIND THE IMPLANT DOSE REQUIRED TO SET $V_T = 0.7V$

IF:

$N_A = 10^{15}/cm^2$

$N_x = 150\, \text{Å}$

$V_{FB} = -0.4V$

USE:

$T = 27^\circ C$

$n_i = 1.45 \times 10^{10}/cm^2$

$E_{ox} = 3.9\, \text{Eo}$, $E_s = 11.8\, \text{Eo}$

$\text{Eo} = 8.854 \times 10^{-14}\, \text{J/cm}^2$

SINCE:

$V_T = \frac{1}{C_{ox}} \left[ 2qE_{ox}N_A(\phi_F) \right] + 2\phi_F + V_{FB} + \frac{qD_i}{C_{ox}}$

1ST CALCULATE A FEW KEY VALUES:

"Thermal Voltage" $= \frac{\Delta T}{q} = \frac{(1.38 \times 10^{-23})(27+273)}{1.602 \times 10^{-19}} = 25.84\, \text{mV}$

$2\phi_F = \Delta T \ln \left( \frac{N_A}{n_i} \right) = (25.84\, \text{mV}) \ln \left( \frac{10^{15}}{1.45 \times 10^{10}} \right) = 575.85\, \text{mV}$

$C_{ox} = \frac{E_{ox}}{E_s} = \frac{3.9 \times (8.854 \times 10^{-14})}{150 \times 10^{-8}} = 230.2 \times 10^{-9}\, \text{F/cm}^2$

NOW, CALCULATE THE 1ST TERM IN THE $V_T$ EQUATION:

$\frac{2qE_{ox}N_A(\phi_F)}{C_{ox}} = \left( \frac{2(1.602 \times 10^{-19})(1.38)(8.854 \times 10^{-14})(10^{15})(57585)}{230.2 \times 10^{-9}} \right)^{1/2}$

$= 60.31\, \text{mV}$

$\Rightarrow V_T = 0.06021 + 575.85 - 0.4 + \frac{qD_i}{C_{ox}} = 0.7V$

$\Rightarrow \frac{qD_i}{C_{ox}} = 0.4638\ \Rightarrow D_i = \frac{(0.4638)(230.2 \times 10^{-9})}{(1.602 \times 10^{-19})}$

$\Rightarrow D_i = 6.67 \times 10^{10}/cm^2$
Now, this gives a value of $V_T = 0.7V$ for $V_{SB} = 0$

**How does $V_T$ vary with $V_{SB}$? (Body effect)**

Recall: $V_T = V_{T0} + \gamma (\sqrt{2} |\phi| + V_{SB} - \sqrt{2} |\phi|)$

$$\gamma = \frac{2qE_S N_A}{Cox}$$

$$\Rightarrow \gamma = \frac{\left[2\left(1.602 \times 10^{-19}\right)(11.8)(8.854 \times 10^{-14})(10^{15})\right]^{1/2}}{230.2 \times 10^{-9}} = 0.8$$

$$\Rightarrow V_T = 0.7 + (0.08)\left(\sqrt{5.7585 + V_{SB}}\right) - \left(\sqrt{5.7585}\right)$$

Some values can be computed as follows:

<table>
<thead>
<tr>
<th>$V_{SB}$ (V)</th>
<th>$V_T$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.70</td>
</tr>
<tr>
<td>1</td>
<td>0.74</td>
</tr>
<tr>
<td>2</td>
<td>0.77</td>
</tr>
<tr>
<td>3</td>
<td>0.79</td>
</tr>
</tbody>
</table>

The function is plotted in Figure 6.16, which illustrates the characteristic square root dependence.

**Figure 6.16** Body-bias effect

**Note:** Numbers shown represent a ~ 0.8\(\mu\)m process. For deep sub-micron processes (e.g., 0.25\(\mu\)m, 0.13\(\mu\)m), body effect goes up!
ASIDE: DIMENSIONAL ANALYSIS

WHEN NOT SURE OF YOUR UNITS, CHECK THEM!

EXAMPLE:

\[ V_T = \frac{1}{C_{ox}} \sqrt{2q\cdot E_s\cdot N_A\cdot \left(2\cdot \phi_0\right)^3 + 2\cdot \phi_0} + V_{FB} + \frac{q\cdot D_s}{C_{ox}} \]

\( \Rightarrow \) EACH TERM HAS UNITS OF VOLTS

NOW, \( C_{ox} = \frac{E_{ox}}{\varepsilon_{ox}} = \frac{(F/cm)}{cm} = \frac{F}{cm^2} \)

\( \Rightarrow \) FOR THE 1ST TERM:

\[ \left( \frac{F}{cm^2} \right)^{-1} \left[ \frac{C}{\left( \frac{F}{cm} \right)} \left( \frac{cm^2}{v} \right) \right]^{1/2} \]

SINCE: \( \varepsilon = C V \Rightarrow C = F V \)

\( \Rightarrow \left( \frac{F}{cm^2} \right)^{-1} \left[ \left( FV \right) \left( \frac{cm^2}{v} \right) \left( \frac{cm^2}{v} \right) \right]^{1/2} \]

\[ = \left( \frac{F}{cm^2} \right)^{-1} \left[ \frac{F^2 V^2}{cm^4} \right]^{1/2} = \left( \frac{cm^2}{F} \right) \left( \frac{FV}{cm^2} \right) = V \checkmark \ CHECK! \]

AND, FOR THE LAST TERM:

\[ \left( \frac{C}{\left( \frac{F}{cm^2} \right)} \right) = \frac{C}{F} = V \checkmark \ CHECK! \]
Derivation of the I-V Equations (Triode Region)

- \( V_{DS} \) applied from drain-to-source causes an electric field as shown above, causing electrons to flow from source to drain.

\[ E(y) = -\ \frac{dV}{dy} \quad \text{with}\quad V(0) = \Phi \]
\[ V(L) = V_{DS} \]

- Since the channel voltage, \( V(y) \), varies along the length of the channel, so does the charge in the channel:

\[ Q(y) = -\text{Cox} \left[ (V_{GS} - V(y)) - V_T \right] \]

"Gate-to-channel" voltage

\[ Q(0) = -\text{Cox} \left( V_{GS} - V_T \right) \quad \text{(source end)} \]
\[ Q(L) = -\text{Cox} \left( V_{GS} - V_{DS} - V_T \right) \quad \text{(drain end)} \]

\[ = V_{GS} \]

Note: Since \( Q(y) \) varies with \( y \), we already know that the equation for \( I_D \) will be non-linear!
Now, to develop the equation for \( I_D \), consider the differential "sheet of charge" shown below:

Figure 9.18 Channel geometry

The voltage across this section \( dy \) wide is:

\[
\frac{dv}{dy} = I_D \frac{dy}{R}
\]

where:

\[
R = \frac{dy}{CN A}
\]

And:

\[
CN = q \mu MN
\]

\[
A = WXE
\]

\[
x_e = \text{channel thickness as a function of } y
\]

\[
\Rightarrow \frac{dv}{dy} = \frac{I_D \frac{dy}{CN WXE}}{q \mu MN WXE}
\]

And since \( Q_e = -qNxe = C_X (V_{GS} - V - V_T) \)

\[
\Rightarrow \frac{dv}{dy} = -\frac{I_D \frac{dy}{CN WQE}}{W_X E (V_{GS} - V - V_T) C_X}
\]
\[ I^0 = \frac{1}{2} m_0 \cos \left( \frac{\theta}{2} \right) \left[ (V_s - V_x)(V_s - V_y) - \frac{(V_x - V_y)^2}{2} \right] \]

\[ I^0 = \frac{1}{2} m_0 \cos \left( \frac{\theta}{2} \right) \left[ (V_s - V_x)(V_s - V_y) - \frac{(V_x - V_y)^2}{2} \right] \]

**Note:** Equations 6.61, 6.62 in text are wrong.

**For saturation substitution:** \( V_s = V_s - V_m = V_s - V_t \)

\[ 2 \]

**If \( y' = \frac{y}{y_s} \) and \( \phi = \frac{\phi}{\phi_s} \):**

\[ \int \frac{I_d}{I^0} = \int m_0 \cos \left( \frac{\theta}{2} \right) \left[ (V_s - V_x)(V_s - V_y) - \frac{(V_x - V_y)^2}{2} \right] \]

\[ \int \frac{V_d}{V_s} = \int m_0 \cos \left( \frac{\theta}{2} \right) \left[ (V_s - V_x)(V_s - V_y) - \frac{(V_x - V_y)^2}{2} \right] \]

\[ \int \frac{V_x}{V_s} = \int m_0 \cos \left( \frac{\theta}{2} \right) \left[ (V_s - V_x)(V_s - V_y) - \frac{(V_x - V_y)^2}{2} \right] \]

**Integration along the length of the channel:**

\[ \frac{I_d}{I^0} = \frac{V_d}{V_s} \frac{V_x}{V_s} = \frac{m_0 \cos \left( \frac{\theta}{2} \right) \left[ (V_s - V_x)(V_s - V_y) - \frac{(V_x - V_y)^2}{2} \right]}{m_0 \cos \left( \frac{\theta}{2} \right) \left[ (V_s - V_x)(V_s - V_y) - \frac{(V_x - V_y)^2}{2} \right]} \]
Example: Find $I_D$ in the circuit below for:

1. $V_{PS} = 2.5\, V$
2. $V_{PS} = 1\, V$

![Circuit Diagram]

Use:

- $W = 2.5\, \mu m$
- $L = 0.25\, \mu m$
- $V_T = 0.7\, V$
- $N = 300\, \text{cm}^2\, /\, \text{V}\cdot\text{s}$
- $X_{ox} = 50\, \text{A}$

G) Since $V_{PS} = 2.5\, V \Rightarrow V_{DS} > V_{GS} - V_T \Rightarrow \text{SATURATION}$

Now, $C_{ox} = \frac{\epsilon_{ox}}{X_{ox}} = \frac{3.9 \times (8.85 \times 10^{-14})}{5 \times 10^{-9}} = 690.6 \times 10^{-9}\, \text{F}/\text{cm}^2$

$$I_D = \frac{1}{2} \left( \frac{W}{L} \right) (V_{GS} - V_T)^2$$

where: $L' = \mu \cdot C_{ox} = 2.07\, \text{mA}/\text{V}^2$

$$I_D = \frac{1}{2} \left( 20\, \text{mA}/\text{V} \right) \left( \frac{2.5}{0.25} \right) (2.5 - 0.7)^2$$

$$I_D = 3.36\, \text{mA}$$

H) With $V_{PS} = 1\, V \Rightarrow V_{PS} < V_{GS} - V_T \Rightarrow \text{TRIODE}$

$$I_D = \frac{1}{2} \left( \frac{W}{L} \right) \left[ (V_{GS} - V_T)(V_{PS}) - \frac{(V_{PS})^2}{2} \right]$$

$$= \left( 20\, \text{mA}/\text{V} \right) \left( \frac{2.5}{0.25} \right) \left[ (2.5 - 0.7)(1) - \frac{1}{2} \right]$$

$$I_D = 2.69\, \text{mA}$$ (Smaller than in (g), as expected)

Unit check: $L' = \mu \cdot C_{ox} = \frac{(\text{cm}^2)}{(\text{cm}^2/\text{V}\cdot\text{s})} = \frac{\text{cm}^2}{\text{V}\cdot\text{s}}$

Since: $Q = CV \Rightarrow F = C/V \Rightarrow L' = \frac{C}{V^2} = \frac{A}{V^2}$ \checkmark
SCALING THEORY

To see how MOSFET performance is affected by scaling the device to smaller dimensions, assume all device sizes are reduced by a scale factor of $s$.

$$
\begin{align*}
W' &= \frac{W}{s} \\
L' &= \frac{L}{s}
\end{align*}
\Rightarrow
\begin{align*}
\frac{W'}{L'} &= \frac{W}{L} = \text{UNCHANGED} \\
A' &= W'L' = \frac{WL}{s^2} = \frac{A}{s^2}
\end{align*}
$$

1. Area drops by $s^2$.
   (E.g., $s=2 \Rightarrow$ only $\frac{1}{4}$ of the area!)

Also, $\frac{C_{ox}'}{C_{ox}} = \frac{E_{ox}'}{E_{ox}} = s \Rightarrow \text{GATE CAPACITANCE GOES UP BY } s$.

And since $g' = \kappa \cdot C_{ox}$, $\frac{g'}{C_{ox}}$ \text{GOES UP BY } s \text{ (ASSUMES } \kappa \text{ CONSTANT)}$

Now, the resistance of an "on" MOS switch (in triode) is given by:

$$
R = \frac{1}{\beta (V_{GS}-V_T)} = \frac{1}{\kappa (V_{GS}-V_T)}
$$

*NOT QUITE TRUE!*

To keep all electric fields constant, scale voltages by $s$ as well, $\Rightarrow$

$$
\begin{align*}
R' &= \frac{1}{\frac{\kappa}{s} (\frac{V_{GS}}{s}-V_T)} = \frac{1}{\frac{\kappa (V_{GS}-V_T)}{s}} = \frac{1}{R} = \text{UNCHANGED}
\end{align*}
\text{AND, } I_D' = s \kappa (\frac{V_{GS}}{s}-V_T) (\frac{V_{DS}}{s}) - \frac{1}{2} (\frac{V_{DS}}{s})^2
\Rightarrow I_D' = \frac{I_D}{s} \Rightarrow \text{CURRENTS SCALE DOWN BY } s
SCALING THEORY (cont)

\[ \text{SINCE \quad \text{POWER} = P = I_D V_{DS} \quad \Rightarrow \quad P' = I_D' V_{DS}' = \left(\frac{I_D}{I_D}ight) \left(\frac{V_{DS}}{V_{DS}}\right) = \frac{P}{S^2} \quad \Rightarrow \quad P' = \frac{P}{S^2} \quad \Rightarrow \quad \text{POWER SCALES DOWN BY} \quad S^2 \]

\[ \text{FINALLY, LOOK AT TRANSIT TIME:} \]

\[ T_x = \frac{L}{V} = \frac{\text{LENGTH}}{\text{VELOCITY}} = \text{TIME REQUIRED FOR A CARRIER TO CROSS THE CHANNEL} \]

\[ \text{\# NOTE: EQUATIONS IN TEXT ARE WRONG! (CHECK UNITS!)} \]

\[ \text{SINCE:} \quad V = \frac{mE}{E} = \frac{V_{DS}}{E} \quad \Rightarrow \quad V = \frac{V_{DS}}{E} \quad \text{(ASSUMES NO V\text{\_sat})} \]

\[ \Rightarrow \quad T_x = \frac{L^2}{mV_{DS}} \]

\[ T_x' = \frac{(L/2)^2}{m\left(V_{DS}/S\right)} = \frac{1}{S} \left(\frac{L^2}{mV_{DS}}\right) = \frac{T_x}{S} \quad \Rightarrow \quad \text{SPEED INCREASES} \]

\[ \text{SUMMARY:} \]

\[ \text{IF ALL DIMENSIONS} \quad (W, L, \tau_{ox}) \quad \text{AND VOLTAGES} \quad (V_{DS}, V_{DS}, V_T) \quad \text{ARE SCALDED DOWN BY} \quad S \quad \Rightarrow \]

- \( \frac{W}{L} = \text{CONSTANT} \)
- \( R = \text{CONSTANT} \)
- \( A \downarrow \text{BY} \quad S^2 \)
- \( E = \text{CONSTANT} \)

- \( I_D, E' \uparrow \text{BY} \quad S \quad \Rightarrow \quad T_x \downarrow \text{BY} \quad S \)

- \( I_D \downarrow \text{BY} \quad S \quad \Rightarrow \quad P \downarrow \text{BY} \quad S^2 \quad \text{BUT, CAN} \quad V_T \quad \text{BE} \quad \text{SCALDED,} \quad S \quad \text{IS} \quad \text{MUST \text{\_CONSTANT?}} \)