The Rail Gun – Basic Design and Principles of Operation

**Rail Gun Design**

The fundamental rail gun design derives from the concept of a linear electromagnetic motor. A conducting bar (the armature) slides along two parallel conducting rails placed in a strong magnetic field. The accelerating force on the armature is produced by current flow through the rails and the armature. Refer to Figure 1 below for a diagram of this conceptual design.

![Figure 1 – Conceptual Design of the Rail Gun](image)

**Analysis of the Electrodynamics**

Consider the forces acting on a single charge carrier ($q$) within the Armature…

![Single charge carrier ($q$) within the Armature](image)
The Lorentz Force Law governs the electrodynamics here:
\[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \]
where
\[ \vec{F} = \text{the force on the charge carrier (N)} \]
\[ \vec{E} = \text{the electric field intensity (V/m)} \]
\[ \vec{v} = \text{the velocity of the charge carrier (m/s)} \]
\[ \vec{B} = \text{the magnetic flux density (T)} \]
This Lorentz Force can be broken down into an electric force \( \vec{F}_e \) and a magnetic force \( \vec{F}_m \), where
\[ \vec{F}_e = q\vec{E} \text{ and } \vec{F}_m = q\vec{v} \times \vec{B}. \]
The initial electric force on \( q \) comes via the electric field intensity produced by the voltage source: \( \vec{E} = \frac{V_{\text{emf}}}{l} \), and therefore \( \vec{F}_e = \frac{qV_{\text{emf}}}{l} \). \( \hat{y} \).
After the bar begins moving along the rails with velocity \( u \), an effective magnetic counter force \( \vec{F}_c \) (often called a “counter emf”) develops:
\[ \vec{F}_c = qu \times B = -quB \hat{y}. \]
The net of these two forces \( \vec{F}_{\text{net}} \) is therefore:
\[ \vec{F}_{\text{net}} = \vec{F}_e + \vec{F}_c = \left( \frac{qV_{\text{emf}}}{l} - quB \right) \hat{y} \] -- (Eq. 1).
The magnetic force accelerating the Armature \( \vec{F}_m \) is a function of the current flow (I) through it and is given by Ampere’s Law as:
\[ \vec{F}_m = I\hat{l} \times \vec{B} = I\hat{y} \times B\hat{z} = I/B\hat{x}. \]
We can also express this force as:
\[ \vec{F}_m = ma = m\frac{du}{dt} \hat{x}. \]
Equating these two we have:
\[ I/B = m\frac{du}{dt} \] -- (Eq. 2).
The current \( I \) at any time during the acceleration of the armature can be given by Ohm’s Law as:
\[ I = \frac{V_{\text{effective}}}{R} \]
where \( V_{\text{effective}} = E_{\text{effective}}/l \) and \( E_{\text{effective}} = F_{\text{net}}/q \). Therefore we can solve for \( I = \frac{F_{\text{net}}/l}{qR} \).
Substituting this value of \( I \) into Eq. 2 and solving for \( F_{\text{net}} \), we get:
\[ F_{\text{net}} = \frac{qRm(l\frac{du}{dt})}{l^2B}. \]
Substituting this value of \( F_{\text{net}} \) into Eq. 1, we get:
\[ \frac{qRm(l\frac{du}{dt})}{l^2B} = \frac{qV_{\text{emf}}}{l} - quB. \]
With a little algebraic rearrangement of this expression, we end up with a first order, inhomogeneous differential equation with constant coefficients:
\[ \frac{du}{dt} + \frac{l^2B^2u}{mR} = \frac{V_{\text{emf}}/B}{mR} \] -- (Eq. 3).
This equation governs the velocity of the bar during acceleration from standstill. As long as all the coefficients in the equation are constants, the analytic solution can be easily found as:
\[ u(t) = \frac{V_{\text{emf}}}{lB} \left[ 1 - e^{-\frac{((l)^2)}{mR}t} \right] \] -- (Eq. 4).

Some questions to ponder here…

1. What physical assumptions and/or simplifications have been made in the analysis here?
2. Why is it important to keep these in mind when trying to build a practical device?
3. How can you find the solution to Eq. 3 if the coefficients are NOT constants?