Problem 5.37  Obtain an expression for the self-inductance per unit length for the parallel wire transmission line of Fig. 5-27(a) in terms of $a$, $d$, and $\mu$, where $a$ is the radius of the wires, $d$ is the axis-to-axis distance between the wires, and $\mu$ is the permeability of the medium in which they reside.

Solution:

![Parallel wire transmission line diagram]

Let us place the two wires in the $x$–$z$ plane and orient the current in one of them to be along the $+z$-direction and the current in the other one to be along the $-z$-direction, as shown in Fig. P5.37. From Eq. (5.30), the magnetic field at point $P = (x, 0, z)$ due to wire 1 is

$$B_1 = \hat{\phi} \frac{\mu I}{2\pi r} = \hat{y} \frac{\mu I}{2\pi x},$$

where the permeability has been generalized from free space to any substance with permeability $\mu$, and it has been recognized that in the $x$–$z$ plane, $\hat{\phi} = \hat{y}$ and $r = x$ as long as $x > 0$. 

Figure P5.37: Parallel wire transmission line.
Given that the current in wire 2 is opposite that in wire 1, the magnetic field created by wire 2 at point $P = (x, 0, z)$ is in the same direction as that created by wire 1, and it is given by

$$B_2 = \hat{y} \frac{\mu I}{2\pi(d-x)}.$$ 

Therefore, the total magnetic field in the region between the wires is

$$B = B_1 + B_2 = \hat{y} \frac{\mu I}{2\pi} \left( \frac{x}{d} + \frac{1}{d-x} \right) = \hat{y} \frac{\mu Id}{2\pi x(d-x)}.$$ 

From Eq. (5.91), the flux crossing the surface area between the wires over a length $l$ of the wire structure is

$$\Phi = \int_S \mathbf{B} \cdot ds = \int_{z=a}^{z=0} \int_{x=a}^{x=d-a} \left( \frac{\mu Id}{2\pi x(d-x)} \right) \cdot (\hat{y} dx dz)$$

$$= \frac{\mu Id}{2\pi} \left( \frac{1}{d} \ln \left( \frac{x}{d-x} \right) \right)\bigg|_{x=a}^{d-a}$$

$$= \frac{\mu Il}{2\pi} \left( \ln \left( \frac{d-a}{a} \right) - \ln \left( \frac{a}{d-a} \right) \right)$$

$$= \frac{\mu Il}{2\pi} \times 2 \ln \left( \frac{d-a}{a} \right) = \frac{\mu Il}{\pi} \ln \left( \frac{d-a}{a} \right).$$

Since the number of ‘turns’ in this structure is 1, Eq. (5.93) states that the flux linkage is the same as magnetic flux: $\Lambda = \Phi$. Then Eq. (5.94) gives a total inductance over the length $l$ as

$$L = \frac{\Lambda}{l} = \frac{\Phi}{l} = \frac{\mu Il}{\pi} \ln \left( \frac{d-a}{a} \right) \quad \text{(H)}.$$ 

Therefore, the inductance per unit length is

$$L' = \frac{L}{l} = \frac{\mu}{\pi} \ln \left( \frac{d-a}{a} \right) \approx \frac{\mu}{\pi} \ln \left( \frac{d}{a} \right) \quad \text{(H/m)},$$

where the last approximation recognizes that the wires are thin compared to the separation distance (i.e., that $d \gg a$). This has been an implied condition from the beginning of this analysis, where the flux passing through the wires themselves have been ignored. This is the thin-wire limit in Table 2-1 for the two wire line.