Exercise 5.24i, Spring 05

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Problem:

1.0 Consider the following state equation:

\[
\dot{X} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u
\]

\[
y = [1 \ 0] X
\]

1.1 u(t) is bounded. Find the ZIR, \( X = \Phi(t)X(0), \) \( X(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \) \( y(t) = [1 \ 1] X, \) and plot \( x_2 \) versus \( x_1 \) for each of the following zeta values indicated in the table below. Indicate in the table below what stability characteristics of the system are implied by the state trajectory plot for each zeta value:

<table>
<thead>
<tr>
<th>zeta</th>
<th>ZIR STABILITY TYPE</th>
<th>ZSR STABILITY TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Asymptotically stable</td>
<td>BIBO Stable</td>
</tr>
<tr>
<td>0.3</td>
<td>Asymptotically stable</td>
<td>BIBO Stable</td>
</tr>
<tr>
<td>0</td>
<td>Marginally stable</td>
<td>Marginally stable</td>
</tr>
<tr>
<td>-0.3</td>
<td>Unstable</td>
<td>Unstable</td>
</tr>
</tbody>
</table>

(Hint: Locate \( X(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) on the plot of \( x_2 \) versus \( x_1. \))

The following m file can be found on voyager/faculty/Heller/241:

```matlab
% chap5labprobpl.m, Prof. MDH 4/15/02. 4/16/05

clear all
format compact
X0=[1;1]; zeta=input('zeta=');
A=[0 1; -1 -2]; zeta=1; eignval=eig(A);
dt=1/(5*max(abs(eignval)));
tend=100/min(abs(eignval));
t=0; dt:tend;
first=length(t); X=[1; y=1];
for k=1:n
    Phi=expm(A.*t(k));
    X(:,k)=Phi*X0;
    pause
end
y=1;1)*X;
figure(1)
c1f
subplot(2,1,1)
plot(X(1,:),X(2,:),'k'
```

The following m file can be found on voyager/faculty/Heller/241:
The Lyapunov Equation defined by the text is $A^T M + M A = \lambda N$; however, the MATLAB function \( M = \text{lyap}(A,B) \) solves for \( M \) where $A^T M + M A = -C$, but, we know from 3.31 that $M = \lambda^{-1}$. Use the Lyapunov equation to determine if your conclusions reached in 1.1 are correct by filling out the following table:

<table>
<thead>
<tr>
<th>zeta</th>
<th>Select $x = \begin{bmatrix} 1 \ 0 \end{bmatrix}$</th>
<th>Eigenvalues of $M$</th>
<th>List the Definiteness of $M$</th>
<th>1.1 Stability Characteristics Confirmed?(No/Yes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\begin{bmatrix} 1 &amp; 2 \ 0 &amp; 1 \end{bmatrix}$</td>
<td>( \lambda_1 = 3.4942 ) ( \lambda_2 = 0.5858 )</td>
<td>Positive definite $\Rightarrow$ Confirmed $\Rightarrow$ Yes (asym. stable)</td>
<td></td>
</tr>
<tr>
<td>0.3</td>
<td>$\begin{bmatrix} 0.505 &amp; 0.651 \ 0.515 &amp; 0.845 \end{bmatrix}$</td>
<td>( \lambda_1 = 0.7724 ) ( \lambda_2 = 4.2776 )</td>
<td>Positive definite $\Rightarrow$ Confirmed $\Rightarrow$ Yes (asym. stable)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>\text{Singular or non-unique}</td>
<td>NA</td>
<td>NA</td>
<td>Not confirmed</td>
</tr>
<tr>
<td>-0.3</td>
<td>$\begin{bmatrix} -0.5505 &amp; 0.165 \ 0.165 &amp; -0.845 \end{bmatrix}$</td>
<td>( \lambda_1 = -0.7724 ) ( \lambda_2 = 4.2776 )</td>
<td>Negative definite $\Rightarrow$ Confirmed $\Rightarrow$ Yes (asym. stable)</td>
<td></td>
</tr>
</tbody>
</table>
2.0 Given \( \dot{X} = AX + Bu \) where

\[
A = \begin{bmatrix}
1 & 3 & 4 \\
2 & 0 & 2 \\
1 & 5 & 5 \\
\end{bmatrix}, \quad A_3 = \begin{bmatrix}
2 & 0 & 2 \\
1 & 0 & 1 \\
1 & 0 & 0 \\
\end{bmatrix}
\]

\( \chi^2 - 2\lambda - 5 \) are in the LHP. \( \chi^3 = 0 \) is a simple root. Marginally stable.

determine the internal stability of the system. (Hint: find Ã is Jordan(A) and minimum polynomial)

Type of Internal Stability = Marginally Stable. Show justification below:

\( A_3 = \begin{bmatrix}
0 & 0 & 0 \\
0 & -2 & 0 \\
0 & 0 & 0 \\
\end{bmatrix} \quad \lambda_3 = 0 \) (repeated)

\( Ã = \begin{bmatrix}
0 & 0 & 0 \\
0 & -2 & 0 \\
0 & 0 & 0 \\
\end{bmatrix} \quad \lambda_1 = -2, \quad \lambda_2 = -5
\]

\( \Delta(\lambda) = \lambda^2 (\lambda + 2) (\lambda + 5) \)

\( \bar{N}_i(\lambda_1) = 1 \quad \bar{N}_i(\lambda_2) = 1 \quad \bar{N}_i(\lambda_3) = 1 \)

\( \bar{N}_i(\lambda_1) = 1 \quad \Rightarrow \quad \psi(\lambda) = \prod_{i=1}^{3} (\lambda - \lambda_i)^{\bar{N}_i} = \lambda (\lambda + 2) (\lambda + 5) \)

Min poly

AND \( \lambda_3 = 0 \) is a simple root of the Min Poly.

\( Ã \) internal stability type is "Marginally Stable".