CHAPTER 2 - CHEN

MATHEMATICAL DESCRIPTIONS OF SYSTEMS

- **Input-Output Acronyms**
  - SISO
  - SIMO
  - MISO
  - MIMO

- **Most systems today are hybrid systems made up of a discrete time controller and a continuous time physical plant (motors, foxes & rabbits, etc.).**

- **$y(t) = 3u(t)$, memory less, $y$ only depends on input**

- **Most systems have memory: system has memory if $y(t)$ depends on $u(t)$ for $t < t_0$, $t = t_0$, and $t > t_0$ past, present, future. That is, there are integrators in the system.**

- Causal or nonanticipatory (all physical systems) if $y(t)$ depends on $u(t)$ for $t < t_0$ and $t = t_0$ past, present

- Anticipatory systems with models are used to find optimal noise filters. Anticipatory systems are not discussed in this course.

- **No energy is stored in a system at $t = t_0$ if the initial state, $x(t = t_0) = 0$.**
DEFINITION 2.1: THE STATE AT $t = t_0$, $X(t_0)$ AND THE INPUT, $U(t)$, FOR $t \geq t_0$ UNIQUELY DETERMINE THE OUTPUT, $Y(t)$, FOR $t \geq t_0$.

The state $X(t_0)$ has memorized the effect of past inputs. The state variables make up the state of the system:

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

- A "DISTRIBUTED" system, such as a transmission line, has an infinite number of state vars.
- Systems with a finite number of state variables are called "LUMPED" systems, the focus for this class will be lumped systems.

**LINEAR SYSTEMS**

- If a system is not linear, it is, by definition, nonlinear. Fortunately, many most nonlinear systems can be approximated as linear systems and still achieve reasonable results.

All systems are nonlinear!
A system is linear if
\[ x_1(t_0) \text{ (initial cond.)} \] \[ \Rightarrow \quad y_1(t), \ t \geq t_0 \]
\[ u_1(t), \ t \geq t_0 \]
AND
\[ x_2(t_0) \text{ (initial cond.)} \] \[ \Rightarrow \quad y_2(t), \ t \geq t_0 \]
\[ u_2(t), \ t \geq t_0 \]
WHERE \( x_1, x_2 \in \mathbb{R}^n \) (any pair of initial conditions)
AND \( u_1 \) and \( u_2 \) are any pair of bounded inputs
WE FIND:
\[ x(t) = x_1(t_0) + x_2(t_0) \] \[ \Rightarrow \quad y(t) = y_1(t) + y_2(t), \ t \geq t_0 \]
\[ u(t) = u_1(t) + u_2(t), \ t \geq t_0 \]
\[ x(0) \text{ (initial state)} \] \[ \Rightarrow \quad y(t) = \alpha x(t), \ t \geq t_0 \]
\[ u(t) = \alpha u(t), \ t \geq t_0 \]
\[ \{ \text{Additivity} \} \]
\[ \{ \text{Property} \} \]
AND
\[ x(t_0) = \alpha x_1(t_0) \] \[ \Rightarrow \quad y(t) = \alpha y(t), \ t \geq t_0 \]
\[ u(t) = \alpha u(t), \ t \geq t_0 \]
\[ \{ \text{Homogeneity} \} \]
OR
\[ x(t_0) = \alpha x_1(t_0) + \alpha x_2(t_0) \] \[ \Rightarrow \quad y(t) = \alpha y_1(t) + \alpha y_2(t), \ t \geq t_0 \]
\[ u(t) = \alpha u(t), \ t \geq t_0 \]
\[ \{ \text{Superposition} \} \]
\[ \{ \text{Property} \} \]
IF A SYSTEM IS NOT LINEAR, THEN IT IS NONLINEAR.

\[ 0 \]
ZIR: ZERO-INPUT RESPONSE
\[ x(t_0) \text{ (initial state)} \] \[ \Rightarrow \quad y_{ZIR}(t), \ t \geq t_0 \]
\[ u(t) = 0, \ t \geq t_0 \]
THE SYSTEM IS ONLY EXCITED BY THE
INITIAL STATE (INITIAL CONDITIONS)
- **ZSR: ZERO-STATE RESPONSE**
  \[
  X(t_0) = 0 \\
  u(t), \ t \geq t_0 \\
  u(t) \neq 0
  \]

- **TOTAL LINEAR SYSTEM RESPONSE**
  APPLYING THE "ADITIVITY PROPERTY"

  \[
  y_{\text{total}}(t) = y_{\text{ZIR}}(t) + y_{\text{ZSR}}(t)
  \]

  **NOT TRUE FOR NONLINEAR SYSTEMS**

- **ZSR I-O DESCRIPTION** \( (X(t_0) = 0) \)

  \[
  y_{\text{ZIR}}(t) = \int_{t_0}^{t} g(t, \tau) u(\tau) \, d\tau, \ t \geq t_0
  \]

  LTV SYSTEM

  FOR LUMPED AND DISTRIBUTED SYSTEMS GIVEN THAT THE INITIAL CONDITIONS ARE ZERO.

  ![Diagram of impulse response](image)

  \[
  u(t) \approx \sum_{n=0}^{\infty} \left[ u(t_n) \right] \frac{\Delta}{n+1}
  \]
Applying the homogeneity property:

\[ x(t) = 0 \]

\[ \sum_{t_2} (t - t_2) u(t) \Delta \rightarrow \text{system} \rightarrow \int_{t_0}^{t_1} (t, t) u(t) \Delta \]

Applying the additivity property:

\[ u(t) = \sum_{t_2} [u(t_2) \Delta (t - t_2)] \rightarrow \text{system} \rightarrow \hat{u}(t) = \sum_{t_2} \int_{t_2}^{t} g(t, t_2) u(t) \Delta \]

\[ u(t) = \lim_{\Delta \rightarrow 0} u(t) \]

implies

\[ y(t) = \lim_{\Delta \rightarrow 0} \hat{u}(t) = \int_{t_0}^{t} g(t, t_2) u(t) \Delta \]

Since we assume causality \( \implies g(t, t_2) = 0 \) \( t < t_2 \)

and a relaxed system, \( x(t_2) = 0 \), \( t_2 \geq t_0 \)

If we have \( p \) inputs and \( q \) outputs

\[ y(t) = \int_{t_0}^{t} g(t, t_2) u(t) \Delta \]

Where

\[ G(t, t_2) \]

is the impulse response at \( u_0 \) if an impulse is applied to \( u_2 \)

\[ G(t, t_2) = \begin{bmatrix} g_{11}(t, t_2) & g_{12}(t, t_2) & \cdots & g_{1p}(t, t_2) \\ g_{21}(t, t_2) & g_{22}(t, t_2) & \cdots & g_{2p}(t, t_2) \\ \vdots & \vdots & \ddots & \vdots \\ g_{q1}(t, t_2) & g_{q2}(t, t_2) & \cdots & g_{qp}(t, t_2) \end{bmatrix} \]

impulse response matrix