**Hough Transform**

**Binary to Lines**

\[ y = mx + c \]

\( x = 1, y = 2 \) \implies 2 = m \cdot 1 + c \implies c = 2 - m \]

**Parameter Space**

\( y = mx + c \)

\( x = 2, y = 3 \) \implies 3 = m \cdot 2 + c \implies c = 3 - 2m \)

\( x = 1, y = 2 \) \implies 2 = m \cdot 1 + c \implies c = 2 - m \)

**Solution**

\( c = 1, m = 1 \) \implies y = x + 1 \)
ROUGH TRANSFORM STEPS - $Y = MX+C$

1. SELECT AXIS SO THAT VERTICAL LINES ARE AVOIDED ($M=\infty$).


3. SET UP AN EQUATION FOR EACH PIXEL IN THE IMAGE YOU WANT TO FIT A LINE TO. THAT IS, $Y = MX+C$ IMPLIES $C = Y - MX$:

$$C = y_1 - MX_1, \ C = y_2 - MX_2, \ C = y_3 - MX_3, \ \text{etc.}$$

4. PLOT EACH OF THE LINES IN THE PARAMETER SPACE YOU FOUND IN 3. TO DO THIS ARBITRARILY SELECT $M$ VALUES, CALCULATE $C$ AND PLOT THE $(M, C)$ POINTS FOR ONE OF THE LINES. USE A STRAIGHT EDGE AND DRAW A STRAIGHT LINE. DO THIS FOR ALL EQUATIONS (ALL POINTS).

5. COUNT THE NUMBER OF LINES CROSSING IN A GRID CELL (ACCUMULATOR CELL) AND PICK THAT NUMBER IN THAT CELL. DO THIS FOR ALL CELLS.

6. THE CELL THAT HAS THE HIGHEST COUNT REPRESENTS THE SOLUTION, I.E., A MVC.

7. IF THERE ARE ADDITIONAL CELLS WITH THE MAXIMUM COUNT, AN ADDITIONAL CRITERION IS REQUIRED.
4. 25 pts. Given the following three points, use the Hough transform technique to determine the line segment(s) which represent a portion of the boundary of an object. You are to use the accumulator grid below. The accumulator cells are 2 units by 2 units square. The center of the accumulator cell represents the a,b value to be used.

[Diagram showing various points and lines with equations and values]
**ROUGH TRANSFORM EXAMPLE**

<table>
<thead>
<tr>
<th>PT #</th>
<th>(x, y)</th>
<th>C = y - Mx</th>
<th>C = y - Mx</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(2, 1)</td>
<td>1 - 2M</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(3, 0)</td>
<td>0 - 2M</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(3, 1)</td>
<td>1 - 3M</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6, 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5, 4)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Solution #1**  
C = 1, M = 0.1

**Solution #2**  
C = -1, M = 1.1

**Solution #3** (? may not be correct)  
C = -0.2, M = 0.7
NORMAL FORM

\[ x_1 \cos(\theta) + y_2 \sin(\theta) = \rho \]

AVOID \( M = \infty \)

\[ y = \left( \frac{-\cos(\theta)}{\sin(\theta)} \right) x + \frac{\rho}{\sin(\theta)} \]

\[ \frac{\rho}{\sin(\theta)} \]

**Figure 3.32** (a) Normal representation of a line. (b) Quantization of the \( \rho\theta \) plane into cells.
-90° ≤ θ ≤ +90°
-D√2 ≤ ρ ≤ +D√2

D = distance between corners in image

Reflective Property (See A,B,C at +/-90°)

Line thru 1,3,5
ρ = 0, θ = -45°

Line thru 2,3,4
ρ = (1.414/2)D, θ = +45°

Each pt is generates a different sine wave
FIGURE 10.21
(a) Infrared image.
(b) Thresholded gradient image.
(c) Hough transform.
(d) Linked pixels.
(Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)