GUIDELINE FOR SKETCHING
THE ROOT LOCUS (RL)

1. NUMBER OF RL BRANCHES = NUMBER OF FINITE CLOSER-LOOP POLES.

2. THE RL IS SYMMETRICAL ABOUT THE REAL AXIS.

3. REAL AXIS SEGMENTS
   FOR K>0, A POINT ON THE REAL AXIS IS
   A POINT ON A BRANCH OF THE ROOT-LOCUS
   IF THE NUMBER OF
   \( \text{FINITE G\_H POLES} \) AND
   \( \text{ZEROS ON THE REAL AXIS TO THE RIGHT OF} \)
   \( \text{THE POINT IS ODD}. \)

4. THE ROOT LOCUS FOR K>0 BEGINS AT THE
   \( \text{FINITE AND INFINITE OPEN-LOOP POLES OF} \)
   G\_H AND ENDS AT THE \( \text{FINITE AND INFINITE} \)
   OPEN-LOOP ZEROS OF G\_H.

5. THE ROOT LOCUS APPROACHES STRAIGHT
   LINE ASYMPTOTES AS THE RL BRANCHES \( \to \infty \)

THE REAL-AXIS INTERCEPT (ASYMPTOTES CROSSING)
K>0,  \( \Theta_a = \text{FINITE G\_H POLES} - \text{FINITE G\_H ZEROS} \)
\#FINITE G\_H POLES - \#FINITE G\_H ZEROS

ANGLE OF ASYMPTOTE WITH REAL-AXIS
K>0,  \( \Theta_a = \frac{(2R+1)\pi}{(\text{\#FINITE G\_H POLES - \#FINITE G\_H ZEROS})} \)

SLOPE = tan \( \Theta_a = \frac{(2R+1)\pi}{(\text{\#FINITE G\_H POLES - \#FINITE G\_H ZEROS})} \), R = 0, 1, 2, ...
5, (CONTINUED)

The number of asymptotes = \(|\text{# finite GH poles} - \text{# finite GH zeros}|\)

6. **CALCULATE BREAKAWAY AND BREAK-IN POINTS**

(NOTE: THERE MAY NOT BE SUCH POINTS.)

![Breakout Point Break-In Point]

**Breakout Point** 2 Equal Closed-Loop Real Poles

Since, breakaway and break-in poles (points) are real, we set \(s = 0\) and write:

\[ 1 + K G(s) H(s) = 0 \]

\[ K = -\frac{1}{G(s) H(s)} = -\frac{D_G(s) D_H(s)}{N_G(s) N_H(s)} \]

For \(K\) values along the real axis, \(K\) is max or min at the breakaway/break-in points on the real branches; hence,

\[ \frac{\partial K}{\partial s} = 0 \] yields the breakaway/break-in points, \(s\)'s.

**OR VIA THE TRANSITION METHOD, SOLVE FOR \(s\)'s (BREAKAWAY/BREAK-IN POINTS)**

\[ \sum_{x=1}^{M} \left( \frac{1}{s - p_x} \right) = \sum_{j=1}^{M} \left( \frac{1}{s - z_j} \right) \]

Where the \(p_x\) and \(z_j\) are the poles and zeros of GH.
7. If there are \( \text{JW crossings} \) a row of the Routh table can be forced to all zeros. The gain \( K \) and the JW closed-loop poles can be solved from the polynomial created by using the coefficients above the row of all zeros in the Routh table.

Note: There may not be JW crossings.

8. Angles of Departure and Arrival

\( \phi_A \) Angle of Departure from Poles

\[ \phi_0 = 180^\circ + \frac{\text{G}H(s-P)}{s=\pi} \]

\( \phi_A \) Angle of Arrival to Zeros

\[ \phi^\circ = 180^\circ - \frac{\text{G}H(s-Z)}{s=\xi} \]


\[ K = \frac{1}{\text{MCH}P} \]
Figure 8.11
System for Example 8.2

\[ \dot{R}(s) + \frac{K(s + 3)}{s(s + 1)(s + 2)(s + 4)} \rightarrow C(s) \]
Figure 8.12
Root locus and asymptotes for the system of Figure 8.11

\[ k_{GH} = \frac{K(5+3)}{5(5+1)(5+2)(5+4)} \]

1. # RL Branches = 4 + 4 = 8
2. RL is symmetric about real axes
3. Find real RL branches
   (000 # real Poles to right)
4. Open-loop poles go to zeros, we have: 3 zeros at \( \infty \)

5. Asymptotes
   High poles
   \[ \text{Asympt.} = 4 - 1 = 3 \]
   High zeros
   \[ \text{Asympt.} = 5 - 1 = 4 \]

\[ \alpha = \frac{(0+1+2+4)}{3} = -\frac{4}{3} = -1.33 \]
\[ \theta_a = \frac{(2\pi+1)\pi}{3} \quad \phi = \alpha \]
\[ 0 \quad \frac{\pi}{3} (60^\circ) \]
\[ 1 \quad \pi \]
\[ -1 \quad -\frac{\pi}{3} (60^\circ) \]
\[ +2 \quad \frac{5\pi}{3} (60^\circ) \]
\[ +3 \quad \frac{2\pi}{3} (60^\circ) \]

\[ k > 9.65 \text{ system is unstable (2 closed-loop poles in RHP) } \]

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6. Breakaway Point, \( \frac{\partial k}{\partial \delta} = -\frac{2 (G \delta + H)}{\delta} \)

\[
\frac{\partial k}{\partial \delta} = -2 \left( \frac{[5(0+1)(0+2)(0+4)]}{(0+3)} \right) = -2 \left( \frac{(5^4 + 75^3 + 140^2 + 8)}{(0+3)} \right)
\]

\[
\frac{\partial k}{\partial \delta} = \left[ (5+3)(45^3 + 215^2 + 285) - (5^4 + 75^3 + 140^2 + 8) \right] \left( \frac{1}{(5+3)^2} \right)
\]

\[
\frac{\partial k}{\partial \delta} = -\left( \frac{30^4 + 260^3 + 775^2 + 840 + 24}{(5+3)^2} \right) = 0
\]

\[30^4 + 260^3 + 775^2 + 840 + 24 = 0\]

\( \sigma_1 = -0.435 \)  Breakaway Point

\( \sigma_2 = -1.610 \)  Not valid, \( \delta \) must be on a RL branch

\( \sigma_3 = -3.31102 + j0.681 \)  Not valid, \( \delta \) must be real

\( \sigma_4 = -3.31102 - j0.681 \)  Not valid, \( \delta \) must be real
7. JW CROSSINGS

\[ T(k) = \frac{k(5+3)}{5^4 + 75^3 + 145^2 + (8+k)5 + 3k} \]

### Table 8.3
Routh table for Eq. (8.40)

<table>
<thead>
<tr>
<th>( s^4 )</th>
<th>1</th>
<th>14</th>
<th>3K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s^3 )</td>
<td>7</td>
<td>8 + K</td>
<td></td>
</tr>
<tr>
<td>( s^2 )</td>
<td>90 - K</td>
<td>21K</td>
<td></td>
</tr>
<tr>
<td>( s^1 )</td>
<td>( \frac{-K^2 - 65K + 720}{90 - K} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s^0 )</td>
<td>21K</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This row can be forced to zeros

\(-K^2 - 65K + 720 = 0\)

\(K = 9.65\)

Row \( s^2 \) polynomial

\((90-K)s^2 + 21K = 0\)

\(80.355^2 + 20217 = 0\)

\(s = \pm j 1.59\)

System is asymptotically stable for \(0 < K < 9.65\)

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\[ K_{GH} = \frac{K (s-3)(s-5)}{(s+1)(s+2)} = \frac{K(s^2-8s+15)}{(s^2+3s+2)} \]

Chapter 8: Root Locus Techniques

\[ H(s) = \frac{K}{s^2-8s+15} \]

\[ 1 + K_{GH} = 0 \]

\[ K = \frac{1}{H(s)} = \frac{1}{(s^2-8s+15)} \]

\[ \frac{\partial K}{\partial \delta} = \frac{-2(\delta^2-26\delta-6)}{(\delta^2-8\delta+15)^2} = 0 \]

\[ \delta_1 = -1.45 \]

\[ \delta_2 = 3.82 \]

\[ \frac{1}{\delta-(3)} + \frac{1}{\delta-(5)} = \frac{1}{\delta-(1)} + \frac{1}{\delta-(2)} \]

\[ 11\delta^2-26\delta-6 = 0 \] (Same as numerator of \( \frac{\partial K}{\partial \delta} \) above)

\[ \delta_1 = -1.45 \]

\[ \delta_2 = 3.82 \]

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Table 8.2
Data for breakaway and break-in points for the root locus of Figure 8.13

<table>
<thead>
<tr>
<th>Real axis value</th>
<th>Gain</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.41</td>
<td>0.008557</td>
<td></td>
</tr>
<tr>
<td>-1.42</td>
<td>0.008585</td>
<td></td>
</tr>
<tr>
<td>-1.43</td>
<td>0.008605</td>
<td></td>
</tr>
<tr>
<td>-1.44</td>
<td>0.008617</td>
<td></td>
</tr>
<tr>
<td>$\delta_1 = -1.45$</td>
<td>0.008623</td>
<td>Max. gain: breakaway</td>
</tr>
<tr>
<td>-1.46</td>
<td>0.008622</td>
<td></td>
</tr>
<tr>
<td>3.3</td>
<td>44.686</td>
<td></td>
</tr>
<tr>
<td>3.4</td>
<td>37.125</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>33.000</td>
<td></td>
</tr>
<tr>
<td>3.6</td>
<td>30.667</td>
<td></td>
</tr>
<tr>
<td>3.7</td>
<td>29.440</td>
<td></td>
</tr>
<tr>
<td>$\delta_2 = 3.8$</td>
<td>29.000</td>
<td>Min. gain: break-in</td>
</tr>
<tr>
<td>3.9</td>
<td>29.202</td>
<td></td>
</tr>
</tbody>
</table>