Figure 5.9
Block diagram for Example 5.1
Figure 5.10
Steps in solving Example 5.1:
(a) collapse summing junctions;
(b) form equivalent cascaded system in the forward path and equivalent parallel system in the feedback path;
(c) form equivalent feedback system and multiply by cascaded $G_1(s)$
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Figure 5.11
Block diagram for Example 5.2
Figure 5.12
Steps in the block diagram reduction for Example 5.2
Finding transient response

Problem: For the system shown in Figure 5.15, find the peak time, percent overshoot, and settling time.

Solution: The closed-loop transfer function found from Eq. (5.9) is

\[ T(s) = \frac{25}{s^2 + 5s + 25} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \] (5.13)

From Eq. (4.18),

\[ \omega_n = \sqrt{25} = 5 \] (5.14)

From Eq. (4.21),

\[ 2\zeta\omega_n = 5 \] (5.15)

Substituting Eq. (5.14) into (5.15) and solving for \( \zeta \) yields

\[ \zeta = 0.5 \] (5.16)

Using the values for \( \zeta \) and \( \omega_n \) along with Eqs. (4.34), (4.38), and (4.42), we find, respectively,

\[ T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = 0.726 \text{ second} \] (5.17)

\[ \%\text{OS} = e^{-\xi\pi} \sqrt{1 - \zeta^2} \times 100 = 16.303 \% \] (5.18)

\[ T_s = \frac{4}{\zeta\omega_n} = 1.6 \text{ seconds} \] (5.19)
**EXAMPLE 5.4**

**Gain design for transient response**

**Problem** Design the value of gain, $K$, for the feedback control system of Figure 5.16 so that the system will respond with a 10% overshoot.

**Solution** The closed-loop transfer function of the system is

$$T(s) = \frac{K}{s^2 + 5s + K} \quad (5.20)$$

![Block diagram](image)

From Eq. (5.20),

$$2\zeta \omega_n = 5$$

and

$$\omega_n = \sqrt{K}$$

Thus

$$\zeta = \frac{5}{2\sqrt{K}} \quad S = \frac{-\ln \left( \frac{\% \text{ overshoot}}{100} \right)}{\pi^2 + \ln^2 (\% \text{ overshoot})}$$

Since percent overshoot is a function only of $\zeta$, Eq. (5.23) shows that the percent overshoot is a function of $K$.

A 10% overshoot implies that $\zeta = 0.591$. Substituting this value for the damping ratio into Eq. (5.23) and solving for $K$ yields

$$K = 17.9$$

Although we are able to design for percent overshoot in this problem, we have not selected settling time as a design criterion because, regardless of the value of $K$, the real parts, $-2.5$, of the poles of Eq. (5.20) remain the same.
Figure 5.17
Signal-flow graph components:

a. system;
b. signal;
c. interconnection of systems and signals

\[ C_1 = (R_1 G_1 - R_2 G_2 + R_3 G_3) G_f \]
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Figure 5.18
Building signal-flow graphs:

a. cascaded system nodes (from Figure 5.3(a));
b. cascaded system signal-flow graph;
c. parallel system nodes (from Figure 5.5(a));
d. parallel system signal-flow graph;
e. feedback system nodes (from Figure 5.6(b));
f. feedback system signal-flow graph