Chapter 4
TIME RESPONSE

Chapter Objectives

In this chapter you will learn the following:

- How to find the time response from the transfer function
- How to use poles and zeros to determine the response of a control system
- How to describe quantitatively the transient response of first- and second-order systems
- How to approximate higher-order systems as first or second-order
- How to view the effects of nonlinearities on the system time response
- How to find the time response from the state-space representation

\[
\frac{C(s)}{R(s)} = \frac{(s+2)}{(s+5)} \quad \text{LET} \quad R(s) = \frac{1}{s} \quad \text{STEP INPUT}
\]

\[
C(s) = \frac{(s+2)}{(s+5)} R(s) = \frac{(s+2)}{(s+5)} \cdot \frac{1}{s} = \frac{(s+2)}{(s+5)} \cdot \frac{1}{s}
\]

**Using a Partial Fraction Expansion:**

\[
C(s) = \frac{(s+2)}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5} = \frac{2.5}{s} + \frac{3/5}{s+5}
\]

where

\[
A = \frac{(s+2)}{(s+5)} \bigg|_{s=0} = \frac{2}{5}
\]

\[
B = \frac{(s+2)}{s} \bigg|_{s=-5} = \frac{3}{5}
\]

Thus,

\[
c(t) = \frac{2}{5} + \frac{3}{5}e^{-5t}
\]
Figure 4.1
a. System showing input and output;
b. pole-zero plot of the system;
c. evolution of a system response.
Follow blue arrows to see the evolution of the response component generated by the pole or zero.
Figure 4.4

a. First-order system;
b. pole plot
Chapter 4: Time Response

\[ C(s) = \frac{a}{s + a(s)} = \frac{1}{s + 1(s)} \]

\[ c(t) = L^{-1}(C(s)) = (1 - e^{-\frac{t}{\tau}})u(t) = (1 - e^{-at})u(t) \]

Figure 4.5
First-order system response to a unit step

\[ c(t) \approx \frac{3}{2} \]

63% of final value at \( t = \text{one time constant} = \tau \)

Initial slope = \( \frac{1}{\text{time constant}} = a \)

Within ±2% of final value

\[ c(t = \tau) = (1 - e^{-1})u(t) \]

\[ = (1 - 0.37)u(t) \]

\[ = 0.63u(t) \]

Value of output at one time constant

\[ \tau = \frac{1}{a} \]

\[ \frac{2}{a} \]

\[ \frac{3}{a} \]

\[ \frac{4}{a} \]

\[ \frac{5}{a} \]

\( t \) (sec)

Rise time \( T_r \)

Settling time \( T_s \)
The rise time \( T_r \) is the time for the waveform (output) to go from 0.1 to 0.9 of its final value. A may not be 1.

\[
T_r = t_{0.9} - t_{0.1}
\]

- Steady-state value (Final value)

\[
\begin{align*}
\frac{1}{K_H} &= 1 \\
C(t_0) &= 0.9 = (1 - e^{-at_0.9}) + 0.1 = e^{-at_0.9} \\
\ln(0.1) &= -at_0.9 \\
\Rightarrow t_0.9 &= \frac{-\ln(0.1)}{a} \\
C(t_0) &= 0.1 = (1 - e^{-at_1}) + 0.9 = e^{-at_1} \\
\ln(0.9) &= -at_1 \Rightarrow t_1 = \frac{-\ln(0.9)}{a} \\
T_r &= t_{0.9} - t_{0.1} = \frac{-\ln(0.1) - (-\ln(0.9))}{a} \\
T_r &= \ln \left( \frac{0.9}{0.1} \right) = 2.197 = 2.2 \text{ or } 2.27
\end{align*}
\]

The settling time \( T_s \) is the time for the response to reach and stay within \( \pm 2\% \) of its final value.

For a 1st order system, \( C = 0.98 = (1 - e^{-\frac{t}{4}}) \)

\[
\Rightarrow \frac{t}{4} = 4 \quad \Rightarrow t = 16
\]
Figure 4.6
Laboratory results of a system step response test

\[
\frac{C(s)}{R(s)} = \frac{K}{s+a}, \quad C(s) = \frac{K}{(s+a)(s)} = \frac{K}{s(5+s)}
\]

\[
C(t = \infty) = 0.72
\]

\[
C(t = 7) = 0.72 * 0.63 = 0.45
\]

\[
\gamma \approx 0.13
\]

\[
\alpha = \frac{\gamma^1}{s+a} = 0.13 = 7.7
\]

\[
\text{Final Value Theorem}
\]

\[
C(t = \infty) = \lim_{s \to 0} s \left[ \frac{K}{s(5+s)} \right] = \frac{K}{5}
\]

\[
K = 0.72, \quad K = 0.72 \times 7.7 = 5.54
\]