Solution

1. 20 pt's. The denominator polynomial of the closed-loop system's transfer function is:

\[ G_{CL}(s) \text{ denominator} = s^3 + (2K+1)s^2 + (-4K+4)s + 4K. \]

a) Using only the Routh Table, determine the range of K for asymptotically stability. A number whose magnitude is less than \(1 \times 10^{-6}\) is to be considered equal to zero.

\[
\begin{array}{c|c|c|c}
  s^3 & 1 & 4-4K \\
  s^2 & 2K+1 & 4K \\
  s^1 & -8K^2+4 & 0 \\
  s^0 & 4K & 0 \\
\end{array}
\]

\[
s = \frac{-1 (4-4K)}{(2K+1) 4K} = \frac{(-4K - (4-4K)(2K+1))}{2K+1}
\]

\[
= -4K + \frac{(-8K^2+4)}{2K+1}
\]

\[a) \ s^2: \ 2K+1 > 0, \ K > -\frac{1}{2} \]

\[s^0: \ 4K > 0, \ K > 0 \]

\[s^1: \ \frac{-8K^2+4}{2K+1} > 0, \ K < \frac{1}{\sqrt{2}} \]

Asymptotic stability range \(0 < K < \frac{1}{\sqrt{2}}\)

b) Marginal stability when \(K = \frac{1}{\sqrt{2}}\)

\[(2K+1)s^2 + 4K = 0\]

\[s^2 = -\frac{4K}{2K+1}\]

\[s = \pm j\sqrt{\frac{4K}{2K+1}} \quad K = \frac{1}{\sqrt{2}}\]
20 pts. Given the following asymptotically stable system:

\[
R \xrightarrow{+} \frac{s^3 + 9s^2 + 23s + 15}{s(s^3 + 12s^2 + 44s + 48)} \xrightarrow{C} \text{TYPE I SYSTEM}
\]

What are the steady-state errors for a step input = 6u(t), a ramp input =6tu(t), and a parabolic input = \(6t^2u(t)\)?

\[
K_0H = \frac{s^3 + 9s^2 + 23s + 15}{s(s^3 + 12s^2 + 44s + 48)} \Rightarrow \text{TYPE I SYSTEM}
\]

Since the Type I system is asymptotically stable,

\[
\epsilon_{ss\text{ step}} = 0
\]

\[
L (6u(t)) = \frac{6}{s} \quad K = 6
\]

\[
\epsilon_{ss\text{ ramp}} = \lim_{{s \to 0}} \frac{K}{s(s^3 + 12s^2 + 44s + 48)} = 19.2
\]

\[
\epsilon_{ss\text{ parabolic}} = \infty
\]
3. 50 pt's. Given the open-loop transfer function, \( KGH = \frac{K}{(s+2.25)(s+3)(s+3)} \), \( K > 0 \):

(a) Design a PI controller so that the system is a Type 1 system by adding a pole and zero. After you have added the pole and zero, sketch the root-locus of the system with a PI controller on the graph provided below. Break-out, break-in, angles of arrival, and angles of departure do not need to be calculated – just estimate them. Break-in or Break-out points are near the intersection of the asymptotes, if there are any. Show in reasonable detail how you arrived at the data you used to sketch your root-locus. Include assumptions made.

(b) Using your root-locus plot below, find the closed-loop poles given the desired OS% = 0.58% as shown on the plot.

(c) Find the gain, \( K \), for the closed-loop poles selected in part b.

\[ K \text{ pole at } s=0 \text{ and a zero at } s=-0.1 \text{ are added. } \]

\[ KGH = \frac{K(s+0.1)}{s(5+2.25)(5+3)(5+3)} \]

\[ \text{Asymptotes: } 3-0=3 \Rightarrow \theta = \frac{-2.25-3-3}{3} = \frac{-8.25}{3} = -2.75 \]

\[ \theta_a = \frac{(20+120)180}{3} = 60^\circ \]

\[ \theta = \frac{(20+120)180}{3} = 180^\circ (\frac{2(2)+120}{3}) = 60^\circ \]

\[ K \alpha \left| \frac{s+2.25)(s+3)}{s+2} \right| \]

\[ \alpha \left| s=-2+0j \right| = 1.91 \]
4. 30 pt's. Given the open-loop transfer function: 
\[ KGH(s) = \frac{\left(\frac{1}{10}s + 1\right)^2}{\left(\frac{1}{20}s + 1\right)^3} \]

(a) Draw the magnitude of the given KGH on the Bode diagram below using the asymptote approach and, then sketch in the actual magnitude.

(b) Approximate the Gain Margin and Phase Margin of the above system using the KGH(j\omega) Bode plot you sketched. Show GM and PM on the Bode plot. 
Record Them Here: \( \text{GM} = +\infty \), \( \text{PM} = +20^\circ \)

(c) Is the system stable? 
 Explain here: \( \text{Stable, GM > 0 and PM > 0} \)

(d) Find the gain \( K \) that will provide a Phase Margin of 65° and find the corresponding Gain Margin. 
Record Them Here: \( K = 20 \), \( \text{GM} = +\infty \), \( \text{PM} = +65^\circ \)