The 360-mm-radius flywheel is rigidly attached to a 30-mm-radius shaft that can roll along parallel rails. Knowing that at the instant shown the center of the shaft has a velocity of 24 mm/s and an acceleration of 10 mm/s², both directed down to the left, determine the acceleration (a) of point A, (b) of point B.

**Velocity analysis.**

Let point G be the center of the shaft and point C be the point of contact with the rails. Point C is the instantaneous center of the wheel and shaft since that point does not slip on the rails.

\[ v_G = r \omega, \quad \omega = \frac{v_G}{r} = \frac{24}{30} = 0.8 \text{ rad/s} \]

**Acceleration analysis.**

Since the shaft does not slip on the rails,

\[ a_C = a_C \downarrow 20^\circ \]

Also,

\[ a_G = \begin{bmatrix} 10 \text{ mm/s}^2 \downarrow \quad 20^\circ \end{bmatrix} \]

\[ a_C = a_G + (a_{CG})_t + (a_{CG})_n \]

\[ \left[ a_C \downarrow 20^\circ \right] = \left[ 10 \text{ mm/s}^2 \downarrow \quad 20^\circ \right] + \left[ 30 \alpha \downarrow 20^\circ \right] + \left[ 30 \alpha^2 \downarrow 20^\circ \right] \]

Components \(20^\circ\): \(10 = -30 \alpha\ \ \ \ \alpha = 0.3333 \text{ rad/s}^2\)

(a) **Acceleration of point A.**

\[ a_A = a_G + (a_{AG})_t + (a_{AG})_n \]

\[ = \begin{bmatrix} 10 \downarrow 20^\circ \quad [360 \alpha \downarrow] + [360 \alpha^2 \downarrow] \end{bmatrix} \]

\[ = [9.3969 \downarrow] + [3.4202 \downarrow] + [120 \downarrow] + [230.4 \downarrow] \]

\[ = [129.3969 \downarrow] + [233.4202 \downarrow] \quad a_A = 267 \text{ mm/s}^2 \downarrow 61.0^\circ \]

(b) **Acceleration of point B.**

\[ a_B = a_G + (a_{BG})_t + (a_{BG})_n \]

\[ = \begin{bmatrix} 10 \downarrow 20^\circ \quad [360 \alpha \downarrow] + [360 \alpha^2 \uparrow] \end{bmatrix} \]

\[ = [9.3969 \downarrow] + [3.4202 \downarrow] + [120 \downarrow] + [230.4 \uparrow] \]

\[ = [110.6031 \downarrow] + [226.9798 \uparrow] \quad a_B = 252 \text{ mm/s}^2 \uparrow 64.0^\circ \]
Arm $AB$ has a constant angular velocity of $16$ rad/s counterclockwise. At the instant when $\theta = 0$, determine the acceleration $(a)$ of collar $D$, $(b)$ of the midpoint $G$ of bar $BD$.

Geometry and velocity analysis.

\[ \theta = 0 \]
\[ v_B = (AB)\omega = (60)(16) = 960 \text{ mm/s} \]
\[ v_B = 960 \text{ mm/s} \uparrow, \quad v_D = v \rightarrow \]

Instantaneous center of bar $BD$ lies at $C$.

\[ \sin \beta = \frac{120}{200} = 0.6, \quad \cos \beta = 0.8 \]
\[ \beta = 36.9^\circ, \quad CB = 200 \cos \beta = 160 \text{ mm} \]
\[ \omega_{BD} = \frac{v_B}{CB} = \frac{960}{160} = 6 \text{ rad/s} \]

\[ \alpha_{AB} = 0 \]

Acceleration analysis.

\[ a_B = [60\alpha_{AB} \uparrow] + [60\omega_{AB}^2 \leftarrow] = 0 + [(60)(16)^2 \leftarrow] = 15360 \text{ mm/s}^2 \leftarrow \]

Point $D$ moves on a straight line.

\[ a_D = a_D \rightarrow \]

\[ (a_{DB})_l = [120\omega_{BD} \uparrow] + [160\alpha_{BD} \uparrow] \]

\[ (a_{DB})_n = [160\omega_{BD}^2 \leftarrow] + [120\omega_{BD}^2 \downarrow] = [5760 \leftarrow] + [4320 \downarrow] \]

\[ a_D = a_B + (a_{DB})_l + (a_{DB})_n. \quad \text{Resolve into components.} \]

\[ + \uparrow: \quad 0 = 0 + 160\alpha_{BD} - 4320 \quad \alpha_{BD} = 27 \text{ rad/s}^2 \]

$(a)$

\[ a_D = -15360 - (120)(27) - 5760 = -24360 \text{ mm/s}^2 \quad a_D = 24.4 \text{ m/s}^2 \rightarrow \]

\[ (a_{GB})_l = [60\omega_{BD} \uparrow] + [80\alpha_{BD} \uparrow] = [1620 \leftarrow] + [2160 \uparrow] \]

\[ (a_{GB})_n = [80\omega_{BD}^2 \leftarrow] + [60\omega_{BD}^2 \downarrow] = [2880 \leftarrow] + [2160 \downarrow] \]

$(b)$

\[ \quad a_G = a_B + (a_{GB})_l + (a_{GB})_n \]
\[ = [15360 \leftarrow] + [1620 \uparrow] + [2160 \uparrow] - [2880 \leftarrow] + [2160 \uparrow] \]
\[ = 19860 \text{ mm/s}^2 \leftarrow \quad a_G = 19.86 \text{ m/s}^2 \rightarrow \]
Chapter 15, Problem 123.

Collar $D$ slides on a fixed vertical rod. Knowing that the disk has a constant angular velocity of 15 rad/s clockwise, determine the angular acceleration of bar $BD$ and the acceleration of collar $D$ when $(a) \theta = 0$, $(b) \theta = 90^\circ$, $(c) \theta = 180^\circ$.

$$\omega_A = 15 \text{ rad/s}, \alpha_A = 0, AB = 2.8 \text{ in.}$$

$$v_B = (AB)\omega_A = (2.8)(15) = 42.0 \text{ in./s}$$

$$a_B = (AB)\omega_A^2 = (2.8)(15)^2 = 630 \text{ in./s}^2$$

(a) $\theta = 0$.

$$v_B = 42 \text{ in./s}, v_D = v_D$$

$$\sin \beta = \frac{2.8}{10} \Rightarrow \beta = 16.26^\circ$$

Instantaneous center of bar $BD$ lies at point $C$.

$$\omega_{BD} = \frac{v_B}{BC} = \frac{42}{10\cos \beta} = 4.375 \text{ rad/s}$$

$$a_B = 630 \text{ in./s}^2, a_D = a_D, \alpha_{BD} = \alpha_{BD}$$

$$\mathbf{a}_{BD} = [10\alpha_{BD} \cos \beta] + [10\alpha_{BD} \sin \beta]$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{BD}$$

Resolve into components.

$$\mathbf{a}_D = 5.58 \text{ rad/s}^2, \mathbf{a}_D = 431 \text{ in./s}^2$$
Chapter 15, Problem 126.

Knowing that at the instant shown bar $AB$ has a constant angular velocity of 4 rad/s clockwise, determine the angular acceleration $(a)$ of bar $BD$. $(b)$ of bar $DE$.

\[ \tan \beta = \frac{60}{120}, \quad \beta = 26.565^\circ, \quad DE = \frac{120}{\cos \beta} = 134.164 \text{ mm} \]

**Velocity analysis.**

\[ \omega_{AB} = 4 \text{ rad/s} \]

\[ v_B = (AB) \omega_{AB} = (200)(4) = 800 \text{ mm/s} \]

\[ v_B = v_B \quad \text{and} \quad v_D = v_D \quad \beta \]

Point $C$ is the instantaneous center of bar $BD$.

\[ BC = \frac{BD}{\tan \beta} = \frac{160}{\tan \beta} = 320 \text{ mm} \]

\[ CD = \frac{BC}{\cos \beta} = 357.77 \text{ mm} \]

\[ \omega_{BD} = \frac{v_B}{BC} = \frac{800}{320} = 2.5 \text{ rad/s} \]

\[ v_D = (CD) \omega_{BD} = (357.77)(2.5) = 894.425 \text{ mm/s} \]

\[ \omega_{DE} = \frac{v_D}{DE} = \frac{894.425}{134.164} = 6.6667 \text{ rad/s} \]

**Acceleration analysis.**

\[ \alpha_{AB} = 0, \quad \alpha_{AB} = 4 \text{ rad/s} \]

\[ a_B = [(AB) \alpha_{AB} \rightarrow] + [(AB) \omega_{AB}^2 \uparrow] \]

\[ = 0 + [(200)(4)^2 \uparrow] = 3200 \text{ mm/s}^2 \uparrow \]