The outer gear C rotates with an angular velocity of 5 rad/s clockwise. Knowing that the inner gear A is stationary, determine (a) the angular velocity of the intermediate gear B, (b) the angular velocity of the arm AB.

Label the contact point between gears A and B as 1, the center of gear B as 2, and the contact point between gears B and C as 3.

Gear A: \[ v_1 = 80\omega_A \] (1)

Arm AB: \[ v_2 = 120\omega_{AB} \] (2)

Gear B: \[ v_1 = v_2 - 40\omega_B \] (3)
\[ v_3 = v_2 + 80\omega_B \] (4)

Gear C: \[ v_3 = 200\omega_C \] (5)

Data: \[ \omega_A = 0, \omega_C = 5 \text{ rad/s} \]

From (1), \[ v_1 = 0, \]

From (5), \[ v_3 = (200)(5) = 1000 \text{ mm/s} \]

From (3), \[ v_2 - 40\omega_B = 0 \] (6)

From (4), \[ v_2 + 80\omega_B = 1000 \] (7)

Solving (6) and (7) simultaneously,

(a) \[ \omega_B = \frac{1000}{120} = 8.333 \text{ rad/s} \]
\[ v_2 = (40)(8.333) = 333.33 \text{ mm/s} \]

(b) From (2), \[ \omega_{AB} = \frac{333.33}{120} = 2.78 \text{ rad/s} \]
\[ \omega_{AB} = 2.78 \text{ rad/s} \]
Chapter 15, Problem 51.

In the planetary gear system shown, the radius of gears $A$, $B$, $C$, and $D$ is 60 mm and the radius of the outer gear $E$ is 180 mm. Knowing that gear $E$ has an angular velocity of 120 rpm clockwise and that the central gear has an angular velocity of 150 rpm clockwise, determine (a) the angular velocity of each planetary gear, (b) the angular velocity of the spider connecting the planetary gears.

Let $a$ be the radius of the central gear $A$, and let $b$ be the radius of the planetary gears $B$, $C$, and $D$. The radius of the outer gear $E$ is $a + 2b$.

Label the contact point between gears $A$ and $B$ as 1, the center of gear $B$ as 2, and the contact point between gears $B$ and $E$ as 3.

Gear $A$:
\[ v_1 = a\omega_A \]  \hfill (1)

Spider:
\[ v_2 = (a + b)\omega_S \]  \hfill (2)

Gear $B$:
\[ v_2 = v_1 + b\omega_B \]  \hfill (3)
\[ v_3 = v_2 + b\omega_B \]  \hfill (4)

Gear $E$:
\[ v_3 = (a + 2b)\omega_E \]  \hfill (5)

From (4) and (5),
\[ v_2 + b\omega_B = (a + 2b)\omega_E \]  \hfill (6)

From (1) and (3),
\[ v_2 - b\omega_B = v_1 = a\omega_A \]  \hfill (7)
Solving for \( v_2 \) and \( \omega_B \),

\[
v_2 = \frac{\left( (a + 2b)\omega_E + a\omega_A \right)}{2}
\]

\[
\omega_B = \frac{\left( (a + 2b)\omega_E - a\omega_A \right)}{2b}
\]

From (2),

\[
\omega_S = \frac{v_2}{a + b}, \quad \omega_S = \frac{\left( (a + 2b)\omega_E + a\omega_A \right)}{2(a + b)}
\]

Data: \( a = 60 \text{ mm} \), \( b = 60 \text{ mm} \), \( a + 2b = 180 \text{ mm} \), \( a + b = 120 \text{ mm} \)

(a)

\[
\omega_B = \frac{180\omega_E - 60\omega_A}{(2)(60)} = 1.5\omega_E - 0.5\omega_A
\]

\[
= (1.5)(120) - (0.5)(150) = 105 \text{ rpm}
\]

\( \omega_B = 105.0 \text{ rpm} \)

(b)

\[
\omega_S = \frac{180\omega_E + 60\omega_A}{(2)(120)} = 0.75\omega_E + 0.25\omega_A
\]

\[
= (0.75)(120) + (0.25)(150) = 127.5 \text{ rpm}
\]

\( \omega_S = 127.5 \text{ rpm} \)
Chapter 15, Problem 61.

In the engine system shown, \( l = 8 \) in. and \( b = 3 \) in. Knowing that the crank \( AB \) rotates with a constant angular velocity of 1000 rpm clockwise, determine the velocity of piston \( P \) and the angular velocity of the connecting rod when (a) \( \theta = 0^\circ \), (b) \( \theta = 90^\circ \).

\[ \omega_{AB} = 1000 \text{ rpm} = \frac{(1000)(2\pi)}{60} = 104.72 \text{ rad/s} \]

(a) \( \theta = 0^\circ \). Crank \( AB \). (Rotation about \( A \)) \( r_{B/A} = 3 \) in.↑

\[ \mathbf{v}_B = r_{B/A} \omega_{AB} = (3)(104.72) = 314.16 \text{ in/s} \rightarrow \]

**Rod BD.** (Plane motion = Translation with \( B \) + Rotation about \( B \))

\[ \mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{DB} \]

\[ v_D^\uparrow = \begin{bmatrix} 314.16 \rightarrow \end{bmatrix} + \begin{bmatrix} v_{DB} \rightarrow \end{bmatrix} \]

\[ v_D = 0, \quad v_{DB} = 314.16 \text{ in/s} \]

\[ \mathbf{v}_P = \mathbf{v}_D \]

\[ v_P = 0 \]

\[ \omega_{BD} = \frac{v_B}{l} = \frac{314.16}{8} = 39.3 \text{ rad/s} \]

(b) \( \theta = 90^\circ \). Crank \( AB \). (Rotation about \( A \)) \( r_{B/A} = 3 \) in.→

\[ \mathbf{v}_B = r_{B/A} \omega_{AB} = (3)(104.72) = 314.16 \text{ in/s} \downarrow \]

**Rod BD.** (Plane motion = Translation with \( B \) + Rotation about \( B \))

\[ \mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{DB} \]

\[ v_D = 314.16 \downarrow + \begin{bmatrix} v_{DB} \downarrow \beta \end{bmatrix} \]

\[ v_{DB} = 0, \quad v_D = 314.16 \text{ in/s} \]

\[ \omega_{BD} = \frac{v_{DB}}{l}, \quad \omega_{BD} = 0 \]

\[ \mathbf{v}_P = \mathbf{v}_D \]

\[ v_P = 314.16 \text{ in/s} \downarrow \quad v_P = 26.2 \text{ ft/s} \downarrow \]
Chapter 15, Problem 63.

In the position shown, bar \( AB \) has an angular velocity of \( 4 \text{ rad/s} \) clockwise. Determine the angular velocity of bars \( BD \) and \( DE \).

\[ \mathbf{v}_B = \omega_{AB} \times \mathbf{r}_{BE} = (-4\mathbf{k}) \times (-0.25\mathbf{j}) = -(1.00 \text{ m/s}) \mathbf{i} \]

\[ \mathbf{v}_D = \omega_{DE} \mathbf{k} \times \mathbf{r}_{DE} = \omega_{DE} \mathbf{k} \times (-0.075\mathbf{i} - 0.15\mathbf{j}) = 0.15\omega_{DE} \mathbf{i} - 0.075\omega_{DE} \mathbf{j} \]

\[ \mathbf{v}_{DB} = \omega_{BD} \mathbf{k} \times \mathbf{r}_{DB} = \omega_{BD} \mathbf{k} \times 0.2\mathbf{i} = 0.2\omega_{BD} \mathbf{j} \]

\[ \mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{DB} \]

\[ 0.15\omega_{DE} \mathbf{i} - 0.075\omega_{DE} \mathbf{j} = -1.00 \mathbf{i} + 0.2\omega_{BD} \mathbf{j} \]

Components:

\[ \mathbf{i}: \quad 0.15\omega_{DE} = -1.00, \quad \omega_{DE} = -6.6667 \text{ rad/s} \quad \omega_{DE} = 6.67 \text{ rad/s} \]

\[ \mathbf{j}: \quad -0.075\omega_{DE} = 0.2\omega_{BD} \]

\[ \omega_{BD} = \frac{(-0.075)(-6.667)}{0.2} = 2.625 \text{ rad/s} \quad \omega_{BD} = 2.50 \text{ rad/s} \]