The uniform rod $AB$ of weight $W$ is released from rest when $\beta = 70^\circ$. Assuming that the friction force is zero between end $A$ and the surface, determine immediately after release (a) the angular acceleration of the rod, (b) the acceleration of the mass center of the rod, (c) the reaction at $A$.

**Solving Problems on Your Own**

1. **Kinematics:** Express the acceleration of the center of mass of the body, and the angular acceleration.

2. **Kinetics:** Draw a free body diagram showing the applied forces and an effective force diagram showing the vector $ma$ or its components and the couple $\vec{I} \alpha$.

3. **Write three equations of motion:** Three equations of motion can be obtained by equating the $x$ components, $y$ components, and moments about an arbitrary point.
**Problem 16.158 Solution**

**Kinematics:**

\[ \omega = 0 \]

\[ \beta = \alpha \sin 70^\circ \]

\[ \omega = 0 \]

\[ (a_{GA})_r = \alpha r_{GA} = \alpha \frac{L}{2} \]

\[ a_G = a_A + a_{GA} \]

\[ a_G = -a_A \mathbf{i} + \alpha \frac{L}{2} \sin 70^\circ \mathbf{i} - \alpha \frac{L}{2} \cos 70^\circ \mathbf{j} \]

\[ a_G = (-a_A + \alpha \frac{L}{2} \sin 70^\circ) \mathbf{i} - \alpha \frac{L}{2} \cos 70^\circ \mathbf{j} \]
Problem 16.158 Solution

Kinetics; draw a free body diagram.

\[ \ddot{\alpha} = \frac{1}{12} mL^2 \alpha \]

\[ m a_{Gx} = m (-a_A + \alpha \frac{L}{2} \sin 70^\circ) \]

\[ m a_{Gy} = -m \alpha \frac{L}{2} \cos 70^\circ \]

Write equations of motion.

Moments about point \( P \) \((+ \beta)\):

\[ mg \left( \frac{L}{2} \cos 70^\circ \right) = m \alpha \frac{L}{2} \cos 70^\circ \left( \frac{L}{2} \cos 70^\circ \right) + \frac{1}{12} mL^2 \alpha \]

\[ \alpha = \frac{6 g \cos 70^\circ}{L \left[ 1 + 3 (\cos 70^\circ)^2 \right]} \]

\[ \alpha = 1.519 \ (g/L) \]
Problem 16.158 Solution

(b) The acceleration of the mass center:
\[ \sum F_x = m a_x: \quad 0 = m (-a_A + \alpha \frac{L}{2} \sin 70^\circ) \]
\[ a_A = \alpha \frac{L}{2} \sin 70^\circ = 1.519 \frac{L}{2} \frac{g}{L} \sin 70^\circ = 0.760 \text{ g} \]
\[ a_G = (-a_A + \alpha \frac{L}{2} \sin 70^\circ) \hat{i} - \alpha \frac{L}{2} \cos 70^\circ \hat{j} \]
Substitute for \( a_A \) and \( \alpha \):
\[ a_G = 0 \hat{i} - 0.260 \text{ g} \hat{j} \]

(b) The reaction at A:
\[ \sum F_y = m a_y: \quad R_A - mg = - m \alpha \frac{L}{2} \cos 70^\circ \]
\[ R_A = mg - m \alpha \frac{L}{2} \cos 70^\circ \]
Substitute for \( \alpha \):
\[ R_A = 0.740 \text{ mg} \]