Problem 15.259

Rod $AB$ of length 125 mm is attached to a vertical rod that rotates about the $y$ axis at the constant rate $\omega_1 = 5 \text{ rad/s}$. Knowing that the angle formed by rod $AB$ and the vertical is increasing at the constant rate $d\beta/dt = 3 \text{ rad/s}$, determine the velocity and acceleration of end $B$ of the rod when $\beta = 30^\circ$.

1. To determine the velocity and acceleration of a point of a body rotating about a fixed point:

1a. Determine the angular velocity $\omega$ of the body: The angular velocity $\omega$ with respect to a fixed frame of reference is often obtained by adding two component angular velocities $\omega_1$ and $\omega_2$. 

Problem 15.259

Solving Problems on Your Own

Rod $AB$ of length 125 mm is attached to a vertical rod that rotates about the $y$ axis at the constant rate $\omega_1 = 5 \text{ rad/s}$. Knowing that the angle formed by rod $AB$ and the vertical is increasing at the constant rate $d\beta/dt = 3 \text{ rad/s}$, determine the velocity and acceleration of end $B$ of the rod when $\beta = 30^\circ$. 

1a. Determine the angular velocity $\omega$ of the body: The angular velocity $\omega$ with respect to a fixed frame of reference is often obtained by adding two component angular velocities $\omega_1$ and $\omega_2$. 

Solving Problems on Your Own

Rod $AB$ of length 125 mm is attached to a vertical rod that rotates about the $y$ axis at the constant rate $\omega_1 = 5 \text{ rad/s}$. Knowing that the angle formed by rod $AB$ and the vertical is increasing at the constant rate $d\beta/dt = 3 \text{ rad/s}$, determine the velocity and acceleration of end $B$ of the rod when $\beta = 30^\circ$.

1b. Compute the velocity of a point of the body:
The velocity $v$ of point $B$ in the body is given by:

$$v = \omega \times r$$

where $r$ is the position vector connecting the fixed point $O$ to point $B$.

1c. Determine the angular acceleration $\alpha$ of the body:

$$\alpha = (\ddot{\omega})_{OXYZ} = (\ddot{\omega})_{xyz} + \Omega \times \omega$$

$\alpha$ is the rate of change $(\ddot{\omega})_{OXYZ}$ of the vector $\omega$ w.r.t. a fixed frame of reference $OXYZ$, $(\ddot{\omega})_{xyz}$ is the rate of change of $\omega$ w.r.t. a rotating frame of reference $xyz$, $\Omega$ is the angular velocity of the rotating frame.
Rod $AB$ of length 125 mm is attached to a vertical rod that rotates about the $y$ axis at the constant rate $\omega_1 = 5$ rad/s. Knowing that the angle formed by rod $AB$ and the vertical is increasing at the constant rate $d\beta/dt = 3$ rad/s, determine the velocity and acceleration of end $B$ of the rod when $\beta = 30^\circ$.

1d. Compute the acceleration of a point of a body:

\[ a_B = \alpha \times r_{B/A} + \omega \times (\omega \times r_{B/A}) \]

Where $a_B$ is the acceleration of point $B$, $a_A$ is the (known) acceleration of point $A$, $\alpha$ and $\omega$ are the angular acceleration and angular velocity of the body with respect to a fixed frame of reference, and $r_{B/A}$ is the position vector of $B$ relative to $A$.

**Problem 15.259 Solution**

Determine the angular velocity of the body.

- $\omega_1 = 5 \, \text{j} \, \text{rad/s}$
- $\omega_2 = d\beta/dt = 3 \, \text{rad/s}$, $\omega_2 = 3 \, \text{k} \, \text{rad/s}$
- $\omega = \omega_1 + \omega_2 = 5 \, \text{j} + 3 \, \text{k} \, \text{rad/s}$

Compute the velocity of a point of the body.

The velocity of end $B$:

For $\beta = 30^\circ$ and $r_{B/A} = 0.125$

$\mathbf{r}_{B/A} = 0.125 \sin 30^\circ \mathbf{i} - 0.125 \cos 30^\circ \mathbf{j} = 0.0625 \mathbf{i} - 0.1083 \mathbf{j}$

$\mathbf{v}_B = \omega \times \mathbf{r}_{B/A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 3 \\ 0.0625 & -0.1083 & 0 \end{vmatrix}$

$\mathbf{v}_B = 0.325 \mathbf{i} + 0.188 \mathbf{j} - 0.313 \mathbf{k} \, \text{m/s}$
Determine the angular acceleration $\alpha$ of the body.

Frame $OXYZ$ is fixed.
Frame $oxyz$ is attached to the vertical rod and rotates with constant angular velocity $\omega_1$.

Consequently: $\dot{\omega}_1 = 0$ and $\Omega = \omega_1$

$\alpha = \dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2 = 0 + \dot{\omega}_2$

$\alpha = (\dot{\omega}_2)_{OXYZ} = (\dot{\omega}_2)_{oxyz} + \Omega \times \omega_2$

$\alpha = 0 + \begin{vmatrix} i & j & k \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{vmatrix}$

$\alpha = 15 \mathbf{i} \text{ rad/s}^2$

Compute the acceleration of a point of a body.

The acceleration of end $B$:
Recall: $\omega = 5 \mathbf{j} + 3 \mathbf{k} \text{ rad/s}$

$\alpha = 15 \mathbf{i} \text{ rad/s}^2$

$r_{B/A} = 0.0625 \mathbf{i} - 0.1083 \mathbf{j}$

$v_B = \omega \times r_{B/A} = 0.325 \mathbf{i} + 0.188 \mathbf{j} - 0.313 \mathbf{k} \text{ m/s}$

$a_B = \alpha \times r_{B/A} + \omega \times (\omega \times r_{B/A})$

$a_B = \alpha \times r_{B/A} + \omega \times v_B$

$a_B = \begin{vmatrix} i & j & k \\ 15 & 0 & 0 \\ 0.0625 & -0.108 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 5 & 3 \\ 0.325 & 0.188 & -0.313 \end{vmatrix}$

$a_B = -2.13 \mathbf{i} + 0.97 \mathbf{j} - 3.25 \mathbf{k} \text{ m/s}^2$