Problem 14.115

A railroad car of length $L$ and mass $m_0$ when empty is moving freely on a horizontal track while being loaded with sand from a stationary chute at a rate $\frac{dm}{dt} = q$. Knowing that the car was approaching the chute at a speed $v_0$, determine (a) the mass of the car and its load after the car has cleared the chute, (b) the speed of the car at that time.
A railroad car of length $L$ and mass $m_0$ when empty is moving freely on a horizontal track while being loaded with sand from a stationary chute at a rate $\frac{dm}{dt} = q$. Knowing that the car was approaching the chute at a speed $v_0$, determine (a) the mass of the car and its load after the car has cleared the chute, (b) the speed of the car at that time.

To solve problems involving a variable system of particles, the principle of impulse and momentum is used.
To solve problems involving a variable system of particles, the principle of impulse and momentum is used.

\[(qt)v_1 = 0\]

We consider the system consisting of the mass \(m_0\) of the car and its contents at \(t = 0\) and of the additional mass \(qt\) which falls into the car in the time interval \(t\).

**Conservation of linear momentum in the horizontal direction**

\[m_0v_0 = (m_0 + qt)v\]

\[v = \frac{m_0v_0}{(m_0 + qt)}\]
\( (qt)v_1 = 0 \)

\[
\begin{align*}
(m_0 + qt)v &= m_0v_0 \\
\end{align*}
\]

\[ v = \frac{m_0v_0}{m_0 + qt} \]

Letting \( v = \frac{dx}{dt} \),

\[ v = \frac{m_0v_0}{m_0 + qt} \]

\[ dx = \frac{m_0v_0 \, dt}{m_0 + qt} \]

\[ x = m_0v_0 \int_0^t \frac{dt}{m_0 + qt} \]

\[ x = \frac{m_0v_0}{q} \left[ \ln(m_0 + qt) \right]_0^t = \frac{m_0v_0}{q} \ln \frac{m_0 + qt}{m_0} \]
Problem 14.115 Solution

\[(qt)v_1 = 0\]

Using the exponential form:

\[m_0 + qt = m_0 e^{qx/m_0v_0}\]

where \(m_0 + qt\) represents the mass at time \(t\) and after the car has moved through \(x\).
Problem 14.115 Solution

\[ (qt)v_1 = 0 \]

\[ v = \frac{m_0v_0}{m_0 + qt} \]

(a) making \( x = L \), we obtain the final mass:

\[ m_f = m_0 + qt_f = m_0 e^{qL/m_0v_0} \]

(b) making \( t = t_f \) in the velocity equation we obtain the final velocity:

\[ v = \frac{m_0v_0}{m_0 + qt_f} = \frac{m_0}{m_f} v_0 = v_0 e^{-qL/m_0v_0} \]