The ends of a chain lie in piles at $A$ and $C$. When given an initial speed $v$, the chain keeps moving freely at that speed over the pulley at $B$. Neglecting friction, determine the required value of $h$. 

Problem 14.114
Solving Problems on Your Own

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The motion of a *variable system of particles*, i.e. a system which is continually *gaining or losing particles* or doing both at the same time involves (1) *steady streams of particles* and (2) *systems gaining or losing mass*.

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We apply the principle of impulse and momentum to the portion of the chain of mass \( m \) in motion at \( t + \Delta t \). Let \( L \) be the length and \( m \) be the mass of the portion of the chain in motion at \( t + \Delta t \). Of this portion of chain, an element at \( A \) of length \( \Delta x \) and mass \( \Delta m = (m/L)\Delta x \) is not in motion at time \( t \). (The extra element at \( C \) is not part of the system considered here.)
Equating moments about $O$:

$$+ \quad r(m - \Delta m)v + rmg(h/L)\Delta t = rm(v + \Delta v)$$

$$-(\Delta m)v + mg(h/L)\Delta t = m(\Delta v)$$

Substituting $\Delta m = (m/L)\Delta x$ and dividing by $(m/L)\Delta t$

$$-v \frac{\Delta x}{\Delta t} + gh = L \frac{\Delta v}{\Delta t}$$
Letting $\Delta t \rightarrow 0$, and noting that $(dx/dt) = v$, 

$$gh - v^2 = L \frac{dv}{dt}$$

If the chain is to keep moving at its initial speed, $dv/dt = 0$, and 

$$h = \frac{v^2}{g}$$