EXAMPLE 14.1

The 10-kg block shown in Fig. 14-6a rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force $P = 400$ N pushes the block up the plane $s = 2$ m.

**SOLUTION**

First the free body diagram of the block is drawn in order to account for all the forces that act on the block, Fig. 14-6b.

**Horizontal Force $P$.** Since this force is constant, the work is determined using Eq. 14-2. The result can be calculated as the force times the component of displacement in the direction of the force; i.e.,

$$U_p = 400 \text{ N} \times (2 \text{ m} \cos 30^\circ) = 692.8 \text{ J}$$

or the displacement times the component of force in the direction of displacement, i.e.,

$$U_p = 400 \text{ N} \cos 30^\circ \times (2 \text{ m}) = 692.8 \text{ J}$$


---

**Spring Force $F_s$.** In the initial position the spring is stretched $s_1 = 0.5 \text{ m}$ and in the final position it is stretched $s_2 = 0.5 + 2 = 2.5 \text{ m}$. We require the work to be negative since the force and displacement are in opposite directions. The work of $F_s$ is thus

$$U_s = \frac{1}{2} [30 \text{ N/m}] (2.5 \text{ m})^2 - \frac{1}{2} [30 \text{ N/m}] (0.5 \text{ m})^2 = -90 \text{ J}$$

**Weight $W$.** Since the weight acts in the opposite direction to its vertical displacement, the work is negative; i.e.,

$$U_W = -98.1 \text{ N} (2 \text{ m} \sin 30^\circ) = -98.1 \text{ J}$$

Note that it is also possible to consider the component of weight in the direction of displacement; i.e.,

$$U_W = -(98.1 \sin 30^\circ \text{ N}) \times 2 \text{ m} = -98.1 \text{ J}$$

**Normal Force $N$.** This force does no work since it is always perpendicular to the displacement.

**Total Work.** The work of all the forces when the block is displaced 2 m is thus

$$U_T = 692.8 - 90 - 98.1 = 505 \text{ J}$$

EXAMPLE 14.2

The 3500-lb automobile shown in Fig. 14-10a is traveling down the 10° inclined road at a speed of 20 ft/s. If the driver jams the brakes, causing his wheels to lock, determine how far s the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is \( \mu_k = 0.5 \).

**SOLUTION**

This problem can be solved using the principle of work and energy, since it involves force, velocity, and displacement.

**Work (Free-Body Diagram).** As shown in Fig. 14-10b, the normal force \( N_A \) does no work since it never undergoes displacement along its line of action. The weight, 3500 lb, is displaced \( x \sin 10° \) and does positive work. Why? The frictional force \( F_A \) does both external and internal work when it is thought to undergo a displacement \( x \). This work is negative since it is in the opposite direction to displacement. Applying the equation of equilibrium normal to the road, we have

\[
\sum F_x = 0; \quad N_A - 3500 \cos 10° \text{ lb} = 0 \quad N_A = 3446.8 \text{ lb}
\]

Thus,

\[
F_A = 0.5N_A = 1723.4 \text{ lb}
\]

**Principle of Work and Energy.**

\[
T_1 + \sum U_{1-2} = T_2
\]

\[
\frac{3500 \text{ lb}}{2 \cdot 32.2 \text{ ft/s}^2} \left[ 20 \text{ ft/s} \right]^2 + \{3500 \text{ lb}(x \sin 10°) - (1723.4 \text{ lb})x\} = 0
\]

Solving for \( x \) yields

\[ s = 19.5 \text{ ft} \quad \text{Ans.} \]

**NOTE:** If this problem is solved by using the equation of motion, two steps are involved. First, from the free-body diagram, Fig. 14-10b, the equation of motion is applied along the incline. This yields

\[ +\sum F_x = ma; \quad 3500 \sin 10° \text{ lb} - 1723.4 \text{ lb} = \frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2} \frac{a}{a} = -10.3 \text{ ft/s}^2
\]

Then, using the integrated form of \( a \text{ds} = v \text{d}t \) (kinematics), since \( a \) is constant, we have

\[
\left( +\varepsilon \right) \quad v^2 = v_0^2 + 2a(s - s_0); \quad (0)^2 = (20 \text{ ft/s})^2 + 2(-10.3 \text{ ft/s}^2)(s - 0)
\]

\[ s = 19.5 \text{ ft} \quad \text{Ans.} \]

EXAMPLE 14.3

For a short time the crane in Fig. 14-11a lifts the 2.50-Mg beam with a force of \( F = (28 + 3s^2) \) kN. Determine the speed of the beam when it has risen \( s = 3 \) m. Also, how much time does it take to attain this height starting from rest?

SOLUTION

We can solve part of this problem using the principle of work and energy since it involves force, velocity, and displacement. Kinematics must be used to determine the time.

Work (Free-Body Diagram). As shown on the free-body diagram, Fig. 14-11a, the towing force \( F \) does positive work, which must be determined by integration since this force is a variable. Also, the weight is constant and will do negative work since the displacement is upwards.

Principles of Work and Energy.

\[
T_1 + \int_0^s (28 + 3s^2)(10^3) \, ds - \int_0^s (2.50)(10^3)(9.81) \, ds = \frac{1}{2}(2.50)(10^3)v^2
\]

\[
28(10^3)s + (10^3)s^3 - 24.525(10^3)s = 1.25(10^3)v^2
\]

\[
v = \sqrt{(2.78s + 0.85s^3)}
\]

When \( s = 3 \) m,

\[
v = 5.47 \text{ m/s} \quad \text{Ans.}
\]

Kinematics. Since we were able to express the velocity as a function of displacement, the time can be determined using \( v = ds/dt \). In this case,

\[
(2.78s + 0.85s^3) = \frac{ds}{dt}
\]

\[
t = \int_0^3 \frac{ds}{(2.78s + 0.85s^3)}
\]

The integration can be performed numerically using a pocket calculator. The result is

\[
t = 1.79 \text{ s} \quad \text{Ans.}
\]

NOTE: The acceleration of the beam can be determined by integrating Eq. (1) using \( a \, dv = a \, ds \), or more directly, by applying the equation of motion, \( \Sigma F = ma \).

EXAMPLE 14.4

The platform $P$, shown in Fig. 14–12a, has negligible mass and is tied down so that the 0.4-m-long cords keep a 1-m-long spring compressed 0.6 m when nothing is on the platform. If a 2-kg block is placed on the platform and released from rest after the platform is pushed down 0.1 m, Fig. 14–12b, determine the maximum height $h$ the block rises in the air, measured from the ground.

![Figure 14-12](image)

SOLUTION

Work (Free-Body Diagram). Since the block is released from rest and later reaches its maximum height, the initial and final velocities are zero. The free-body diagram of the block when it is still in contact with the platform is shown in Fig. 14–12c. Note that the weight does negative work and the spring force does positive work. Why? In particular, the initial compression in the spring is $s_1 = 0.6 \text{ m} + 0.1 \text{ m} = 0.7 \text{ m}$. Due to the cords, the spring’s final compression is $s_2 = 0.6 \text{ m}$ (after the block leaves the platform). The bottom of the block rises from a height of $(0.4 \text{ m} - 0.1 \text{ m}) = 0.3 \text{ m}$ to a final height $h$.

Principle of Work and Energy.

$$T_1 + \sum U_{i-2} = T_2$$

$$\frac{1}{2}mv^2 + \{-(\frac{1}{2}ks^2 - \frac{1}{2}ks') - W \Delta x\} = \frac{1}{2}mv^2$$

Note that here $s_1 = 0.7 \text{ m}$ and $s_2 = 0.6 \text{ m}$ and so the work of the spring as determined from Eq. 14–4 will indeed be positive once the calculation is made. Thus,

$$0 + \left\{ \left[\frac{1}{2}(200 \text{ N/m})(0.6 \text{ m})^2 - \frac{1}{2}(200 \text{ N/m})(0.7 \text{ m})^2 \right] - (19.62 \text{ N})(h - (0.3 \text{ m})) \right\} = 0$$

Solving yields

$$h = 0.963 \text{ m} \quad \text{Ans.}$$
EXAMPLE 14.5

Packages having a mass of 2 kg are delivered from a conveyor to a smooth circular ramp with a velocity of \( v_0 = 1 \text{ m/s} \) as shown in Fig. 14-13a. If the radius of the ramp is 0.5 m, determine the angle \( \theta = \theta_{\text{max}} \) at which each package begins to leave the surface.

SOLUTION

**Work (Free-Body Diagram).** The free-body diagram of the block is shown at the intermediate location \( \theta \). The weight \( W = 2(9.81) = 19.62 \text{ N} \) does positive work during the displacement. If a package is assumed to leave the surface when \( \theta = \theta_{\text{max}} \), then the weight moves through a vertical displacement of \( (0.5 - 0.5 \cos \theta_{\text{max}}) \) m, as shown in the figure.

**Principle of Work and Energy:**

\[
T_1 + \Sigma U_{1-2} = T_2
\]

\[
\frac{1}{2}(2 \text{ kg})(1 \text{ m/s})^2 + (19.62 \text{ N}(0.5 - 0.5 \cos \theta_{\text{max}}) \text{ m}) = \frac{1}{2}(2 \text{ kg})v_2^2
\]

\[
v_2^2 = 9.81(1 - \cos \theta_{\text{max}}) + 1 \tag{1}
\]

**Equation of Motion.** There are two unknowns in Eq. 1, \( \theta_{\text{max}} \) and \( v_2 \). A second equation relating these two variables may be obtained by applying the equation of motion in the **normal direction** to the forces on the free-body diagram. (The principle of work and energy has replaced application of \( \Sigma F_i = ma \), as noted in the derivation.) Thus,

\[+v^2 \Sigma F_n = ma_n; \quad -N_B + 19.62 \cos \theta = (2 \text{ kg}) \left( \frac{v^2}{0.5 \text{ m}} \right)\]

When the package leaves the ramp at \( \theta = \theta_{\text{max}} \), \( N_B = 0 \) and \( v = v_2 \); hence, this equation becomes

\[
\cos \theta_{\text{max}} = \frac{v_2^2}{4.905} \tag{2}
\]

Eliminating the unknown \( v_2^2 \) between Eqs. 1 and 2 gives

\[4.905 \cos \theta_{\text{max}} = 9.81(1 - \cos \theta_{\text{max}}) + 1 \]

Solving, we have

\[
\cos \theta_{\text{max}} = 0.735 \quad \theta_{\text{max}} = 42.7^\circ \quad \text{Ans.}
\]

**NOTE:** This problem has also been solved in Example 13.9 using the equation of motion. If the two methods of solution are compared, it will be apparent that a work-energy approach yields a more direct solution.

EXAMPLE 14.6

The blocks A and B shown in Fig. 14–14a have a mass of 10 kg and 100 kg, respectively. Determine the distance B travels from the point where it is released from rest to the point where its speed becomes 2 m/s.

**SOLUTION**

This problem may be solved by considering the blocks separately and applying the principle of work and energy to each block. However, the work of the (unknown) cable tension can be eliminated from the analysis by considering blocks A and B together as a system. The solution will require simultaneous solution of the equations of work and energy and kinematics. To be consistent with our sign convention, we will assume both blocks move in the positive downward direction.

**Work (Free-Body Diagram).** As shown on the free-body diagram of the system, Fig. 14–14b, the cable forces T and reactions R1 and R2 do no work, since these forces represent the reactions at the supports and consequently do not move while the blocks are being displaced. The weights both do positive work since, as stated above, they are both assumed to move downward.

![Image](https://example.com/image.png)

**Fig. 14-14**

---

**Principle of Work and Energy.** Realizing the blocks are released from rest, we have

\[
\Sigma T_1 + \Sigma T_{12} - \Sigma T_2 = \left\{ \frac{1}{2} m_A (v_A)^2 + \frac{1}{2} m_B (v_B)^2 \right\} + \{ W_A \Delta s_A + W_B \Delta s_B \} = \left\{ \frac{1}{2} m_A (v_A)^2 + \frac{1}{2} m_B (v_B)^2 \right\}
\]

\[
\{ 0 + 0 \} + \{ 98.1 \text{ N} (\Delta s_A) + 981 \text{ N} (\Delta s_B) \} = \left\{ \frac{1}{2} (10 \text{ kg})(v_A)^2 + \frac{1}{2} (100 \text{ kg})(2 \text{ m/s})^2 \right\}
\]

(1)

**Kinematics.** Using the methods of kinematics discussed in Sec. 12.9, it may be seen from Fig. 14–14a that at any given instant the total length of all the vertical segments of cable may be expressed in terms of the position coordinates \( x_A \) and \( x_B \) as

\[ x_A + 4 x_B = l \]

Hence, a change in position yields the displacement equation

\[
\Delta x_A + 4 \Delta x_B = 0
\]

\[
\Delta s_A = -4 \Delta s_B
\]

(2)

As required, both of these displacements are positive downward. Taking the time derivative yields

\[ v_A = -4 v_B = -4(2 \text{ m/s}) = -8 \text{ m/s} \]

**Retention** the negative sign in Eq. 2 and substituting into Eq. 1 yields

\[ \Delta s_B = 0.883 \text{ m} \]

\[ *Ans* \]

EXAMPLE 14.7

The motor \( M \) of the hoist shown in Fig. 14–15a operates with an efficiency of \( \epsilon = 0.85 \). Determine the power that must be supplied to the motor to lift the 75-lb crate \( C \) at the instant point \( P \) on the cable has an acceleration of 4 ft/s² and a velocity of 2 ft/s. Neglect the mass of the pulley and cable.

SOLUTION

In order to compute the power output of the motor, it is first necessary to determine the tension in the cable since this force is developed by the motor.

From the free-body diagram, Fig. 14–15b, we have

\[
\sum F_y = ma_y; \quad -2T + 75 lb = \frac{75 lb}{32.2 \text{ ft/s}^2}a_C
\]

(1)

The acceleration of the crate can be obtained by using kinematics to relate it to the known acceleration of point \( P \), Fig. 14–15a. Using the methods of Sec. 12.9, the coordinates \( x_C \) and \( y_C \) in Fig. 14–15a can be related to a constant portion of cable length / which is changing in the vertical and horizontal directions. We have \( 2x_C + x_P = l \). Taking the second time derivative of this equation yields

\[
2a_C = -a_P
\]

(2)

Since \( a_P = +4 \text{ ft/s}^2 \), then \( a_C = (-4 \text{ ft/s}^2)/2 = -2 \text{ ft/s}^2 \). What does the negative sign indicate? Substituting this result into Eq. 1 and retaining the negative sign since the acceleration in both Eqs. 1 and 2 is considered positive downward, we have

\[
-2T + 75 lb = \frac{75 lb}{32.2 \text{ ft/s}^2}(-2 \text{ ft/s}^2)
\]

\[
T = 39.8 \text{ lb}
\]

The power output, measured in units of horsepower, required to draw the cable in at a rate of 2 ft/s is therefore

\[
P = T \cdot v = (39.8 \text{ lb})(2 \text{ ft/s})(1 \text{ hp/(550 ft-lb/s)})
\]

\[
P = 0.145 \text{ hp}
\]

This power output requires that the motor provide a power input of

\[
\text{power input} = \frac{1}{\epsilon} \times (\text{power output})
\]

\[
= \frac{1}{0.85}(0.145 \text{ hp}) = 0.170 \text{ hp}
\]

Ans.

NOTE: Since the velocity of the crate is constantly changing, the power requirement is instantaneous.
**EXAMPLE 14.8**

The sports car shown in Fig. 14-16a has a mass of 2 Mg and is traveling at a speed of 25 m/s, when the brakes to all the wheels are applied. If the coefficient of kinetic friction is $\mu_k = 0.35$, determine the power developed by the friction force when the car skids. Then find the car's speed after it has slid 10 m.

SOLUTION

As shown on the free-body diagram, Fig. 14-16b, the normal force $N_c$ and frictional force $F_c$ represent the resultant forces of all four wheels.

Applying the equation of equilibrium in the y direction to determine $N_c$, we have

$$+ \sum F_y = 0; \quad N_c = 19.62 \text{ kN}$$

The kinetic frictional force is therefore

$$F_c = 0.35(19.62 \text{ kN}) = 6.867 \text{ kN}$$

The velocity of the car can be determined when $s = 10$ m by applying the principle of work and energy. Why?

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(2000 \text{ kg})(25 \text{ m/s})^2 - 6.867(10^3) \text{ N (10 m)} = \frac{1}{2}(2000 \text{ kg})v^2$$

$$v = 23.59 \text{ m/s}$$

The power of the frictional force at this instant is therefore

$$P = |F_c \cdot v| = 6.867(10^3) \text{ N (25 m/s)} = 172 \text{ kW} \quad \text{Ans}$$
**EXAMPLE 14.9**

The gantry structure in the photo is used to test the response of an airplane during a crash. As shown in Fig. 14-21a, the plane, having a mass of 8 Mg, is hoisted back until \( \theta = 60^\circ \), and then the pull-back cable AC is released when the plane is at rest. Determine the speed of the plane just before crashing into the ground, \( \theta = 15^\circ \). Also, what is the maximum tension developed in the supporting cable during the motion? Neglect the effect of lift caused by the wings during the motion and the size of the airplane.

![Diagram](image)

**SOLUTION**

Since the force of the cable does no work on the plane, it must be obtained using the equation of motion. First, however, we must determine the plane's speed at \( B \).

**Potential Energy.** For convenience, the datum has been established at the top of the gantry.

**Conservation of Energy.**

\[
T_A + V_A = T_B + V_B
\]

\[
0 - 8000 \text{ kg} \times (9.81 \text{ m/s}^2) \times (20 \cos 60^\circ \text{ m}) = \]

\[
\left\{ (8000 \text{ kg}) v_B^2 - 8000 \text{ kg} \times (9.81 \text{ m/s}^2) \times (20 \cos 15^\circ \text{ m}) \right\}
\]

\[ v_B = 13.5 \text{ m/s} \]

**Equation of Motion.** Using the data tabulated on the free-body diagram when the plane is at \( B \), Fig. 14-21b, we have

\[ + \sum F_x = ma_x; \]

\[ T - 8000(9.81) \text{ N} \times 15^\circ = (8000 \text{ kg}) \times \left( \frac{(13.5 \text{ m/s})^2}{20 \text{ m}} \right) \]

\[ T = 149 \text{ kN} \]

EXAMPLE 14.10

The ram \( R \) shown in Fig. 14–22a has a mass of 100 kg and is released from rest 0.75 m from the top of a spring, \( A \), that has a stiffness \( k_A = 12 \text{ kN/m} \). If a second spring, \( B \), having a stiffness \( k_B = 15 \text{ kN/m} \), is “nested” in \( A \), determine the maximum displacement of \( A \) needed to stop the downward motion of the ram. The unstretched length of each spring is indicated in the figure. Neglect the mass of the springs.

**SOLUTION**

**Potential Energy.** We will assume that the ram compresses both springs at the instant it comes to rest. The datum is located through the center of gravity of the ram at its initial position, Fig. 14–22b. When the kinetic energy is reduced to zero \( (v_2 = 0) \), \( A \) is compressed a distance \( s_A \) and \( B \) compresses \( s_B = s_A - 0.1 \text{ m} \).

**Conservation of Energy.**

\[
T_1 + V_1 = T_2 + V_2 \]
\[
0 = 0 + \left[ \frac{1}{2} k_A s_A^2 + \frac{1}{2} k_B (s_A - 0.1)^2 \right] - W_R \]
\[
0 = 0 + \left[ \frac{1}{2} (12000 \text{ N/m}) s_A^2 + \frac{1}{2} (15000 \text{ N/m}) (s_A - 0.1 \text{ m})^2 \right] - 981 \text{ N} (0.75 \text{ m} + s_A) \]

Rearranging the terms,
\[
13500 s_A^2 - 2481 s_A - 660.75 = 0 \]

Using the quadratic formula and solving for the positive root, we have

\[ s_A = 0.331 \text{ m} \]

\textbf{Ans.}

Since \( s_A = 0.331 \text{ m} - 0.1 \text{ m} = 0.231 \text{ m} \), which is positive, the assumption that both springs are compressed by the ram is correct.

**NOTE:** The second root, \( s_A = -0.148 \text{ m} \), does not represent the physical situation. Since positive \( s \) is measured downward, the negative sign indicates that spring \( A \) would have to be “extended” by an amount of 0.148 m to stop the ram.

EXAMPLE 14.11

A smooth 2-kg collar C, shown in Fig. 14–23a, fits loosely on the vertical shaft. If the spring is unstretched when the collar is at the position A, determine the speed at which the collar is moving when \( y = 1 \) m, if (a) it is released from rest at A, and (b) it is released at A with an upward velocity \( v_A = 2 \) m/s.

SOLUTION

Part (a) Potential Energy. For convenience, the datum is established through \( AB \), Fig. 14–23b. When the collar is at \( C \), the gravitational potential energy is \( (mg)y \), since the collar is below the datum, and the elastic potential energy is \( \frac{1}{2} kx^2_{AB} \). Here \( x_{AB} = 0.5 \) m, which represents the stretch in the spring as shown in the figure.

Conservation of Energy.

\[
T_A + V_A = T_C + V_C
\]

\[
0 + 0 = \frac{1}{2}mv_C^2 + \left\{ \frac{1}{2} kx_{AB}^2 - mgy \right\}
\]

\[
0 + 0 = \frac{1}{2}(2 \text{ kg})v_C^2 + \left\{ \frac{1}{2}(3 \text{ N/m})(0.5 \text{ m})^2 - 2(9.81) \text{ N}(1 \text{ m}) \right\}
\]

\[
v_C = 4.39 \text{ m/s} \uparrow \quad \text{Ans.}
\]

This problem can also be solved by using the equation of motion or the principle of work and energy. Note that in both of these methods the variation of the magnitude and direction of the spring force must be taken into account (see Example 13.4). Here, however, the above method of solution is clearly advantageous since the calculations depend only on data calculated at the initial and final points of the path.

Part (b) Conservation of Energy. If \( v_A = 2 \) m/s, using the data in Fig. 14–23b, we have

\[
T_A + V_A = T_C + V_C
\]

\[
\frac{1}{2}mv_A^2 + 0 = \frac{1}{2}mv_C^2 + \left\{ \frac{1}{2} kx_{AB}^2 - mgy \right\}
\]

\[
\frac{1}{2}(2 \text{ kg})(2 \text{ m/s})^2 + 0 = \frac{1}{2}(2 \text{ kg})v_C^2 + \left\{ \frac{1}{2}(3 \text{ N/m})(0.5 \text{ m})^2 \right\} - 2(9.81) \text{ N}(1 \text{ m})
\]

\[
v_C = 4.82 \text{ m/s} \uparrow \quad \text{Ans.}
\]

NOTE: The kinetic energy of the collar depends only on the magnitude of velocity, and therefore it is immaterial if the collar is moving up or down at 2 m/s when released at A.