EXAMPLE

Given: A 0.5 kg ball of negligible size is fired up a vertical track of radius 1.5 m using a spring plunger with $k = 500 \text{ N/m}$. The plunger keeps the spring compressed 0.08 m when $s = 0$.

Find: The distance $s$ the plunger must be pulled back and released so the ball will begin to leave the track when $\theta = 135^\circ$.

Plan:
1) Draw the FBD of the ball at $\theta = 135^\circ$.
2) Apply the equation of motion in the n-direction to determine the speed of the ball when it leaves the track.
3) Apply the principle of work and energy to determine $s$.

EXAMPLE (continued)

Solution:
1) Draw the FBD of the ball at $\theta = 135^\circ$.

The weight ($W$) acts downward through the center of the ball. The normal force exerted by the track is perpendicular to the surface. The friction force between the ball and the track has no component in the n-direction.

2) Apply the equation of motion in the n-direction. Since the ball leaves the track at $\theta = 135^\circ$, set $N = 0$.

$=> \sum F_n = ma_n = m \left(\frac{v^2}{\rho}\right) => W \cos 45^\circ = m \left(\frac{v^2}{\rho}\right)$

$=> (0.5)(9.81) \cos 45^\circ = (0.5/1.5)v^2 => v = 3.2257 \text{ m/s}$
EXAMPLE (continued)

3) Apply the principle of work and energy between position 1 ($\theta = 0$) and position 2 ($\theta = 135^\circ$). Note that the normal force ($N$) does no work since it is always perpendicular to the displacement direction. (Students: Draw a FBD to confirm the work forces).

$$T_1 + \sum U_{1-2} = T_2$$

$$0.5m(v_1)^2 - W \Delta y - (0.5k(s_2)^2 - 0.5k(s_1)^2) = 0.5m(v_2)^2$$

and

$$v_1 = 0, \quad v_2 = 3.2257 \text{ m/s}$$

$$s_1 = s + 0.08 \text{ m}, \quad s_2 = 0.08 \text{ m}$$

$$\Delta y = 1.5 + 1.5 \sin 45^\circ = 2.5607 \text{ m}$$

$$=> 0 - (0.5)(9.81)(2.5607) - [0.5(500)(0.08)^2 - 0.5(500)(5 + 0.08)^2] = 0.5(0.5)(3.2257)^2$$

$$=> s = 0.179 \text{ m} = 179 \text{ mm}$$

GROUP PROBLEM SOLVING

Given: Block A has a weight of 60 lb and block B has a weight of 10 lb. The coefficient of kinetic friction between block A and the incline is $\mu_k = 0.2$. Neglect the mass of the cord and pulleys.

Find: The speed of block A after it moves 3 ft down the plane, starting from rest.

Plan: 1) Define the kinematic relationships between the blocks.

2) Draw the FBD of each block.

3) Apply the principle of work and energy to the system of blocks.
GROUP PROBLEM SOLVING
(continued)

Solution:

1) The kinematic relationships can be determined by defining position coordinates \( s_A \) and \( s_B \), and then differentiating.

Since the cable length is constant:

\[
2s_A + s_B = l \\
2\Delta s_A + \Delta s_B = 0 \\
\Delta s_A = \text{3 ft} \implies \Delta s_B = -\text{6 ft}
\]

and

\[
2v_A + v_B = 0 \\
\implies v_B = -2v_A
\]

Note that, by this definition of \( s_A \) and \( s_B \), positive motion for each block is defined as downwards.

GROUP PROBLEM SOLVING
(continued)

2) Draw the FBD of each block.

Sum forces in the y-direction for block A (note that there is no motion in this direction):

\[
\sum F_y = 0: \ N_A - \left(\frac{4}{5}\right)W_A = 0 \implies N_A = \left(\frac{4}{5}\right)W_A
\]
GROUP PROBLEM SOLVING (continued)

3) Apply the principle of work and energy to the system (the blocks start from rest).

\[ \Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2 \]

\[ (0.5m_A(v_{A1})^2 + 0.5m_B(v_{B1})^2) + ((3/5)W_A - 2T - \mu N_A)\Delta s_A + (W_B - T)\Delta s_B = (0.5m_A(v_{A2})^2 + 0.5m_B(v_{B2})^2) \]

\[ v_{A1} = v_{B1} = 0, \Delta s_A = 3\text{ ft}, \Delta s_B = 6\text{ ft}, v_B = -2v_A, N_A = (4/5)W_A \]

\[ => 0 + 0 + (3/5)(60)(3) - 2T(3) - (0.2)(0.8)(60)(3) + (10)(-6) - T(-6) = 0.5(60/32.2)(v_{A2})^2 + 0.5(10/32.2)(-2v_{A2})^2 \]

\[ => v_{A2} = 3.52 \text{ ft/s} \]

Note that the work due to the cable tension force on each block cancels out.

POWER AND EFFICIENCY PROCEDURE FOR ANALYSIS

- Find the resultant external force acting on the body causing its motion. It may be necessary to draw a free-body diagram.

- Determine the velocity of the point on the body at which the force is applied. Energy methods or the equation of motion and appropriate kinematic relations, may be necessary.

- Multiply the force magnitude by the component of velocity acting in the direction of \( F \) to determine the power supplied to the body (\( P = F \cdot v \cdot \cos \theta \)).

- In some cases, power may be found by calculating the work done per unit of time (\( P = dU/dt \)).

- If the mechanical efficiency of a machine is known, either the power input or output can be determined.
Given: A sports car has a mass of 2 Mg and an engine efficiency of $\varepsilon = 0.65$. Moving forward, the wind creates a drag resistance on the car of $F_D = 1.2v^2$ N, where $v$ is the velocity in m/s. The car accelerates at 5 m/s$^2$, starting from rest.

Find: The engine’s input power when $t = 4$ s.

Plan:  
1) Draw a free body diagram of the car.  
2) Apply the equation of motion and kinematic equations to find the car’s velocity at $t = 4$ s.  
3) Determine the power required for this motion.  
4) Use the engine’s efficiency to determine input power.

### EXAMPLE

Solution:  
(continued)

1) Draw the FBD of the car.

The drag force and weight are known forces. The normal force $N_c$ and frictional force $F_c$ represent the resultant forces of all four wheels. The frictional force between the wheels and road pushes the car forward.

2) The equation of motion can be applied in the x-direction, with $a_x = 5$ m/s$^2$:

$$\sum F_x = ma_x \Rightarrow F_c - 1.2v^2 = (2000)(5)$$

$$\Rightarrow F_c = (10,000 + 1.2v^2) \text{ N}$$
3) The constant acceleration equations can be used to determine the car’s velocity.

\[ v_x = v_{xo} + a_x t = 0 + (5)(4) = 20 \text{ m/s} \]

4) The power output of the car is calculated by multiplying the driving (frictional) force and the car’s velocity:

\[ P_o = (F_c)(v_x) = [10,000 + (1.2)(20)^2](20) = 209.6 \text{ kW} \]

5) The power developed by the engine (prior to its frictional losses) is obtained using the efficiency equation.

\[ P_i = \frac{P_o}{\varepsilon} = \frac{209.6}{0.65} = 322 \text{ kW} \]

EXAMPLE (continued)

CONSERVATION OF ENERGY
(Section 14.6)

When a particle is acted upon by a system of conservative forces, the work done by these forces is conserved and the sum of kinetic energy and potential energy remains constant. In other words, as the particle moves, kinetic energy is converted to potential energy and vice versa. This principle is called the principle of conservation of energy and is expressed as

\[ T_1 + V_1 = T_2 + V_2 = \text{Constant} \]

\( T_1 \) stands for the kinetic energy at state 1 and \( V_1 \) is the potential energy function for state 1. \( T_2 \) and \( V_2 \) represent these energy states at state 2. Recall, the kinetic energy is defined as \( T = \frac{1}{2} mv^2 \).
EXAMPLE

Given: The girl and bicycle weigh 125 lbs. She moves from point A to B.

Find: The velocity and the normal force at B if the velocity at A is 10 ft/s and she stops pedaling at A.

Plan: Note that only kinetic energy and potential energy due to gravity ($V_B$) are involved. Determine the velocity at B using the conservation of energy equation and then apply equilibrium equations to find the normal force.

Solution:

Placing the datum at B:

\[
T_A + V_A = T_B + V_B
\]

\[
\frac{1}{2} \left( \frac{125}{32.2} \right)^2 + 125(30) = \frac{1}{2} \left( \frac{125}{32.2} \right) V_B^2
\]

\[
V_B = 45.1 \text{ ft/s}
\]

Equation of motion applied at B:

\[
\sum F_a = m a_n = \frac{m}{\rho} V_B^2
\]

\[
N_B - 125 = \frac{125}{32.2} \left( \frac{45.1}{2} \right)^2
\]

\[
N_B = 283 \text{ lb}
\]
GROUP PROBLEM SOLVING

Given: The mass of the collar is 2 kg and the spring constant is 60 N/m. The collar has no velocity at A and the spring is un-deformed at A.

Find: The maximum distance y the collar drops before it stops at Point C.

Plan: Apply the conservation of energy equation between A and C. Set the gravitational potential energy datum at point A or point C (in this example, choose point A).

GROUP PROBLEM SOLVING

(continued)

Solution:
Notice that the potential energy at C has two parts ($T_c = 0$).

$$V_c = (V_c)_e + (V_c)_g$$

Placing the datum for gravitational potential at A yields a conservation of energy equation with the left side all zeros. Since $T_c$ equals zero at points A and C, the equation becomes

$$0 + 0 = 0 + \left[ \frac{60}{2} \sqrt{(.75)^2 + y^2} - .75)^2 - 2(9.81)y \right]$$

Note that $(V_c)_g$ is negative since point C is below the datum. Since the equation is nonlinear, a numerical solver can be used to find the solution or root of the equation. This solving routine can be done with a calculator or a program like Excel. The solution yields $y = 1.61$ m.
Also notice that since the velocities at $A$ and $C$ are zero, the velocity must reach a maximum somewhere between $A$ and $C$.

Since energy is conserved, the point of maximum kinetic energy (maximum velocity) corresponds to the point of minimum potential energy.

By expressing the potential energy at any given position as a function of $y$ and then differentiating, we can determine the position at which the velocity is maximum (since $dV/dy = 0$ at this position). The derivative yields another nonlinear equation which could be solved using a numerical solver.