**THE WORK OF A FORCE, PRINCIPLE OF WORK AND ENERGY, & PRINCIPLE OF WORK AND ENERGY FOR A SYSTEM OF PARTICLES**

**Today’s Objectives:**
Students will be able to:
1. Calculate the work of a force.
2. Apply the principle of work and energy to a particle or system of particles.

**In-Class Activities:**
- Applications
- Work of A Force
- Principle of Work And Energy
- Group Problem Solving

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**APPLICATIONS**

A roller coaster makes use of gravitational forces to assist the cars in reaching high speeds in the “valleys” of the track.

How can we design the track (e.g., the height, h, and the radius of curvature, ρ) to control the forces experienced by the passengers?
Crash barrels are often used along roadways for crash protection. The barrels absorb the car’s kinetic energy by deforming.

If we know the typical velocity of an oncoming car and the amount of energy that can be absorbed by each barrel, how can we design a crash cushion?

Another equation for working kinetics problems involving particles can be derived by integrating the equation of motion \((F = ma)\) with respect to displacement.

By substituting \(a = \frac{v}{(dv/ds)}\) into \(F = ma\), the result is integrated to yield an equation known as the principle of work and energy.

This principle is useful for solving problems that involve force, velocity, and displacement. It can also be used to explore the concept of power.

To use this principle, we must first understand how to calculate the work of a force.
WORK OF A FORCE
(Section 14.1)

A force does work on a particle when the particle undergoes a displacement along the line of action of the force. Work is defined as the product of force and displacement components acting in the same direction. So, if the angle between the force and displacement vector is \( \theta \), the increment of work \( dU \) done by the force is

\[ dU = F \, ds \cos \theta \]

By using the definition of the dot product and integrating, the total work can be written as

\[ U_{1-2} = \int_{r_1}^{r_2} F \cdot dr \]

WORK OF A FORCE
(continued)

If \( F \) is a function of position (a common case) this becomes

\[ U_{1-2} = \int_{s_1}^{s_2} F \cos \theta \, ds \]

If both \( F \) and \( \theta \) are constant \( (F = F_c) \), this equation further simplifies to

\[ U_{1-2} = F_c \cos \theta \,(s_2 - s_1) \]

Work is positive if the force and the movement are in the same direction. If they are opposing, then the work is negative. If the force and the displacement directions are perpendicular, the work is zero.
WORK OF A WEIGHT

The work done by the gravitational force acting on a particle (or weight of an object) can be calculated by using

\[ U_{1-2} = \int_{y_1}^{y_2} -W \, dy = -W (y_2 - y_1) = -W \Delta y \]

The work of a weight is the product of the magnitude of the particle’s weight and its vertical displacement. If \( \Delta y \) is upward, the work is negative since the weight force always acts downward.

WORK OF A SPRING FORCE

When stretched, a linear elastic spring develops a force of magnitude \( F_s = ks \), where \( k \) is the spring stiffness and \( s \) is the displacement from the unstretched position.

The work of the spring force moving from position \( s_1 \) to position \( s_2 \) is

\[ U_{1-2} = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} k \, s \, ds = 0.5k(s_2)^2 - 0.5k(s_1)^2 \]

If a particle is attached to the spring, the force \( F_s \) exerted on the particle is opposite to that exerted on the spring. Thus, the work done on the particle by the spring force will be negative or

\[ U_{1-2} = - \left[ 0.5k (s_2)^2 - 0.5k (s_1)^2 \right] \]
SPRING FORCES

It is important to note the following about spring forces:

1. The equations just shown are for linear springs only! Recall that a linear spring develops a force according to \( F = ks \) (essentially the equation of a line).

2. The work of a spring is not just spring force times distance at some point, i.e., \( (ks_i)(s_i) \). Beware, this is a trap that students often fall into!

3. Always double check the sign of the spring work after calculating it. It is positive work if the force put on the object by the spring and the movement are in the same direction.

PRINCIPLE OF WORK AND ENERGY

By integrating the equation of motion, \( \sum F_i = ma_i = mv(dv/ds) \), the principle of work and energy can be written as

\[
\sum U_{1-2} = 0.5m(v_f)^2 - 0.5m(v_i)^2 \quad \text{or} \quad T_1 + \sum U_{1-2} = T_2
\]

\( \sum U_{1-2} \) is the work done by all the forces acting on the particle as it moves from point 1 to point 2. Work can be either a positive or negative scalar.

\( T_1 \) and \( T_2 \) are the kinetic energies of the particle at the initial and final position, respectively. Thus, \( T_1 = 0.5 \text{ m } (v_i)^2 \) and \( T_2 = 0.5 \text{ m } (v_f)^2 \). The kinetic energy is always a positive scalar (velocity is squared!).

So, the particle’s initial kinetic energy plus the work done by all the forces acting on the particle as it moves from its initial to final position is equal to the particle’s final kinetic energy.
The principle of work and energy cannot be used, in general, to determine forces directed normal to the path, since these forces do no work.

Both kinetic energy and work have the same units, that of energy! In the SI system, the unit for energy is called a joule (J), where 1 J = 1 N·m. In the FPS system, units are ft·lb.

The principle of work and energy cannot be used, in general, to determine forces directed normal to the path, since these forces do no work.

The principle of work and energy can also be applied to a system of particles by summing the kinetic energies of all particles in the system and the work due to all forces acting on the system.

Today’s Objectives:
Students will be able to:
1. Understand the concept of conservative forces and determine the potential energy of such forces.
2. Apply the principle of conservation of energy.

In-Class Activities:
• Applications
• Conservative Force
• Potential Energy
• Conservation of Energy
• Group Problem Solving
The weight of the sacks resting on this platform causes potential energy to be stored in the supporting springs.

If the sacks weigh 100 lb and the equivalent spring constant is $k = 500 \text{ lb/ft}$, what is the energy stored in the springs?

When a ball of weight $W$ is dropped (from rest) from a height $h$ above the ground, the potential energy stored in the ball is converted to kinetic energy as the ball drops.

What is the velocity of the ball when it hits the ground? Does the weight of the ball affect the final velocity?
CONSERVATIVE FORCE
(Section 14.5)

A force $F$ is said to be conservative if the work done is
independent of the path followed by the force acting on a particle
as it moves from A to B. In other words, the work done by the
force $F$ in a closed path (i.e., from A to B and then back to A)
equals zero.

$$\oint F \cdot dr = 0$$

This means the work is conserved.

A conservative force depends
only on the position of the
particle, and is independent of its
velocity or acceleration.

CONSERVATIVE FORCE
(continued)

A more rigorous definition of a conservative force makes
use of a potential function ($V$) and partial differential
calculus, as explained in the texts. However, even without
the use of the these mathematical relationships, much can be
understood and accomplished.

The “conservative” potential energy of a particle/system is
typically written using the potential function $V$. There are two
major components to $V$ commonly encountered in mechanical
systems, the potential energy from gravity and the potential
energy from springs or other elastic elements.

$$V_{\text{total}} = V_{\text{gravity}} + V_{\text{springs}}$$
POTENTIAL ENERGY

Potential energy is a measure of the amount of work a conservative force will do when it changes position.

In general, for any conservative force system, we can define the potential function (V) as a function of position. The work done by conservative forces as the particle moves equals the change in the value of the potential function (the sum of $V_{\text{gravity}}$ and $V_{\text{springs}}$).

It is important to become familiar with the two types of potential energy and how to calculate their magnitudes.

POTENTIAL ENERGY DUE TO GRAVITY

The potential function (formula) for a gravitational force, e.g., weight ($W = mg$), is the force multiplied by its elevation from a datum. The datum can be defined at any convenient location. The potential function is given by

$$V_g = \pm Wy$$

$V_g$ is positive if $y$ is above the datum and negative if $y$ is below the datum. Remember, YOU get to set the datum.
ELASTIC POTENTIAL ENERGY

Recall that the force of an elastic spring is $F = ks$. It is important to realize that the potential energy of a spring, while it looks similar, is a different formula.

\[ V_e = \frac{1}{2} ks^2 \]

Notice that the potential function $V_e$ always yields positive energy.

CONSERVATION OF ENERGY

(Section 14.6)

When a particle is acted upon by a system of conservative forces, the work done by these forces is conserved and the sum of kinetic energy and potential energy remains constant. In other words, as the particle moves, kinetic energy is converted to potential energy and vice versa. This principle is called the principle of conservation of energy and is expressed as

\[ T_1 + V_1 = T_2 + V_2 = \text{Constant} \]

$T_1$ stands for the kinetic energy at state 1 and $V_1$ is the potential energy function for state 1. $T_2$ and $V_2$ represent these energy states at state 2. Recall, the kinetic energy is defined as $T = \frac{1}{2} mv^2$. 
End of the Lecture

Let Learning Continue