Today’s Objectives:
Students will be able to:
1. Describe the velocity of a rigid body in terms of translation and rotation components.
2. Perform a relative-motion velocity analysis of a point on the body.

In-Class Activities:
- Applications
- Translation and Rotation Components of Velocity
- Relative Velocity Analysis
- Group Problem Solving

APPLICATIONS

As the slider block A moves horizontally to the left with $v_A$, it causes the link CB to rotate counterclockwise. Thus $v_B$ is directed tangent to its circular path.

Which link is undergoing general plane motion?
How can its angular velocity, $\omega$, be found?
Applications (continued)

Planetary gear systems are used in many automobile automatic transmissions. By locking or releasing different gears, this system can operate the car at different speeds. How can we relate the angular velocities of the various gears in the system?

Relative Motion Analysis (Section 16.5)

When a body is subjected to general plane motion, it undergoes a combination of translation and rotation.

Point A is called the base point in this analysis. It generally has a known motion. The x’-y’ frame translates with the body, but does not rotate. The displacement of point B can be written:

\[ \mathbf{dr}_B = \mathbf{dr}_A + \mathbf{dr}_{B/A} \]

Disp. due to translation
Disp. due to translation and rotation
Disp. due to rotation
RELATIVE MOTION ANALYSIS: VELOCITY

The velocity at B is given as: $\frac{dr_B}{dt} = \frac{dr_A}{dt} + \frac{dr_{B/A}}{dt}$ or

$$v_B = v_A + v_{B/A}$$

Since the body is taken as rotating about A,

$$v_{B/A} = \frac{dr_{B/A}}{dt} = \omega \times r_{B/A}$$

Here $\omega$ will only have a $k$ component since the axis of rotation is perpendicular to the plane of translation.

RELATIVE MOTION ANALYSIS: VELOCITY

(continued)

$$v_B = v_A + \omega \times r_{B/A}$$

When using the relative velocity equation, points A and B should generally be points on the body with a known motion. Often these points are pin connections in linkages.

Here both points A and B have circular motion since the disk and link BC move in circular paths. The directions of $v_A$ and $v_B$ are known since they are always tangent to the circular path of motion.
Furthermore, point B at the center of the wheel moves along a horizontal path. Thus, $v_B$ has a known direction, e.g., parallel to the surface.

When a wheel rolls without slipping, point A is often selected to be at the point of contact with the ground. Since there is no slipping, point A has zero velocity.

Furthermore, point B at the center of the wheel moves along a horizontal path. Thus, $v_B$ has a known direction, e.g., parallel to the surface.

**PROCEDURE FOR ANALYSIS**

The relative velocity equation can be applied using either a Cartesian vector analysis or by writing scalar x and y component equations directly.

Scalar Analysis:

1. Establish the fixed x-y coordinate directions and draw a kinematic diagram for the body. Then establish the magnitude and direction of the relative velocity vector $v_{B/A}$.

2. Write the equation $v_B = v_A + \omega \times r_{B/A}$ and by using the kinematic diagram, underneath each term represent the vectors graphically by showing their magnitudes and directions.

3. Write the scalar equations from the x and y components of these graphical representations of the vectors. Solve for the unknowns.
Vector Analysis:

1. Establish the fixed x-y coordinate directions and draw the kinematic diagram of the body, showing the vectors $\mathbf{v}_A$, $\mathbf{v}_B$, $\mathbf{r}_{B/A}$ and $\mathbf{\omega}$. If the magnitudes are unknown, the sense of direction may be assumed.

2. Express the vectors in Cartesian vector form and substitute into $\mathbf{v}_B = \mathbf{v}_A + \mathbf{\omega} \times \mathbf{r}_{B/A}$. Evaluate the cross product and equate respective $i$ and $j$ components to obtain two scalar equations.

3. If the solution yields a negative answer, the sense of direction of the vector is opposite to that assumed.

**PROCEDURE FOR ANALYSIS (continued)**

**EXAMPLE**

**Given:** Block A is moving down at 2 m/s.

**Find:** The velocity of B at the instant $\theta = 45^\circ$.

**Plan:** 1. Establish the fixed x-y directions and draw a kinematic diagram.

2. Express each of the velocity vectors in terms of their $i$, $j$, $k$ components and solve $\mathbf{v}_B = \mathbf{v}_A + \mathbf{\omega} \times \mathbf{r}_{B/A}$. 

![Diagram](image)
EXAMPLE
(continued)

Solution:

\[
\mathbf{v}_B = \mathbf{v}_A + \omega_{AB} \times \mathbf{r}_{B/A}
\]

\[
\mathbf{v}_B = -2 \mathbf{j} + (\omega \mathbf{k} \times (0.2 \sin 45 \mathbf{i} - 0.2 \cos 45 \mathbf{j}))
\]

Equating the \(i\) and \(j\) components gives:

\[
\mathbf{v}_B = 0.2 \omega \cos 45 \\
0 = -2 + 0.2 \omega \sin 45
\]

Solving:

\[
\omega = 14.1 \text{ rad/s} \quad \text{or} \quad \omega_{AB} = 14.1 \text{ rad/s} \mathbf{k}
\]

\[
\mathbf{v}_B = 2 \text{ m/s} \quad \text{or} \quad \mathbf{v}_B = 2 \text{ m/s} \mathbf{i}
\]

EXAMPLE II

Given: Collar C is moving downward with a velocity of 2 m/s.

Find: The angular velocities of CB and AB at this instant.

Plan: Notice that the downward motion of C causes B to move to the right. Also, CB and AB both rotate counterclockwise.

First, draw a kinematic diagram of link CB and use \(\mathbf{v}_B = \mathbf{v}_C + \omega_{CB} \times \mathbf{r}_{B/C}\). (Why do CB first?) Then do a similar process for link AB.
EXAMPLE II

(continued)

**Solution:**

Link CB. Write the relative-velocity equation:

\[ \mathbf{v}_B = v_C + \omega_{CB} \times \mathbf{r}_{B/C} \]

\[ \mathbf{v}_B = -2 \mathbf{j} + \omega_{CB} \mathbf{k} \times (0.2 \mathbf{i} - 0.2 \mathbf{j}) \]

\[ \mathbf{v}_B = -2 \mathbf{j} + 0.2 \omega_{CB} \mathbf{j} + 0.2 \omega_{CB} \mathbf{i} \]

By comparing the \(i\), \(j\) components:

\[ \begin{align*}
    i: & \quad v_B = 0.2 \omega_{CB} \quad \Rightarrow \quad v_B = 2 \text{ m/s } i \\
    j: & \quad 0 = -2 + 0.2 \omega_{CB} \quad \Rightarrow \quad \omega_{CB} = 10 \text{ rad/s } k
\end{align*} \]

EXAMPLE II

(continued)

Link AB experiences only rotation about A. Since \(v_B\) is known, there is only one equation with one unknown to be found.

\[ v_B = \omega_{AB} \times \mathbf{r}_{B/A} \]

\[ 2 \mathbf{i} = \omega_{AB} \mathbf{k} \times (-0.2 \mathbf{j}) \]

\[ 2 \mathbf{i} = 0.2 \omega_{AB} \mathbf{i} \]

By comparing the \(i\)-components:

\[ 2 = 0.2 \omega_{AB} \]

So, \(\omega_{AB} = 10 \text{ rad/s } k\)
GROUP PROBLEM SOLVING

Given: The crankshaft AB is rotating at 500 rad/s about a fixed axis passing through A.

Find: The speed of the piston P at the instant it is in the position shown.

Plan: 1) Draw the kinematic diagram of each link showing all pertinent velocity and position vectors.
      2) Since the motion of link AB is known, apply the relative velocity equation first to this link, then link BC.

Solution:

1) First draw the kinematic diagram of link AB.

   Link AB rotates about a fixed axis at A. Since \( \omega \) is ccw, \( v_B \) will be directed down, so \( v_B = -v_B \hat{j} \).

   Applying the relative velocity equation with \( v_A = 0 \):
   
   \[
   v_B = v_A + \omega \times r_{B/A}
   \]
   
   \[-v_B \hat{j} = (500 \hat{k}) \times (-0.1 \hat{i} + 0 \hat{j})
   \]
   
   \[-v_B \hat{j} = -50 \hat{j} + 0 \hat{i}
   \]

   Equating \( j \) components: \( v_B = 50 \)
   
   \[ v_B = -50 \hat{j} \text{ m/s} \]
2) Now consider link BC. Since point C is attached to the piston, \( v_C \) must be directed up or down. It is assumed here to act down, so \( v_C = -v_Cj \). The unknown sense of \( \omega_{BC} \) is assumed here to be ccw: \( \omega_{BC} = \omega_{BC}k \).

Applying the relative velocity equation:
\[
v_C = v_B + \omega_{BC} \times r_{C/B}
\]
\[
-\dot{v}_C = -50i + (\omega_{BC})k \times (0.5 \cos 60^\circ i + 0.5 \sin 60^\circ j)
\]
\[
-\dot{v}_C = -50i + 0.25\omega_{BC}j - 0.433\omega_{BC}i
\]
\( i : \) \( 0 = -0.433\omega_{BC} \Rightarrow \omega_{BC} = 0 \)
\( j : \) \(-\dot{v}_C = -50 + 0.25\omega_{BC} \Rightarrow v_C = 50 \)
\( v_C = -50j \text{ m/s} \)

---

**INSTANTANEOUS CENTER OF ZERO VELOCITY**

**Today’s Objectives:**
Students will be able to:
1. Locate the instantaneous center of zero velocity.
2. Use the instantaneous center to determine the velocity of any point on a rigid body in general plane motion.

**In-Class Activities:**
- Applications
- Location of the Instantaneous Center
- Velocity Analysis
- Group Problem Solving
The instantaneous center (IC) of zero velocity for this bicycle wheel is at the point in contact with ground. The velocity direction at any point on the rim is perpendicular to the line connecting the point to the IC.

Which point on the wheel has the maximum velocity?

As the board slides down the wall (to the left) it is subjected to general plane motion (both translation and rotation).

Since the directions of the velocities of ends A and B are known, the IC is located as shown.

What is the direction of the velocity of the center of gravity of the board?
INSTANTANEOUS CENTER OF ZERO VELOCITY  
(Section 16-6)

For any body undergoing planar motion, there always exists a point in the plane of motion at which the velocity is instantaneously zero (if it were rigidly connected to the body).

This point is called the instantaneous center of zero velocity, or IC. It may or may not lie on the body!

If the location of this point can be determined, the velocity analysis can be simplified because the body appears to rotate about this point at that instant.

LOCATION OF THE INSTANTANEOUS CENTER

To locate the IC, we can use the fact that the velocity of a point on a body is always perpendicular to the relative position vector from the IC to the point. Several possibilities exist.

First, consider the case when velocity $v_A$ of a point A on the body and the angular velocity $\omega$ of the body are known.

In this case, the IC is located along the line drawn perpendicular to $v_A$ at A, a distance $r_{A/IC} = v_A/\omega$ from A. Note that the IC lies up and to the right of A since $v_A$ must cause a clockwise angular velocity $\omega$ about the IC.
A second case is when the lines of action of two non-parallel velocities, \(v_A\) and \(v_B\), are known.

First, construct line segments from A and B perpendicular to \(v_A\) and \(v_B\). The point of intersection of these two line segments locates the IC of the body.

A third case is when the magnitude and direction of two parallel velocities at A and B are known.

Here the location of the IC is determined by proportional triangles. As a special case, note that if the body is translating only (\(v_A = v_B\)), then the IC would be located at infinity. Then \(\omega\) equals zero, as expected.
VELOCITY ANALYSIS

The velocity of any point on a body undergoing general plane motion can be determined easily once the instantaneous center of zero velocity of the body is located.

Since the body seems to rotate about the IC at any instant, as shown in this kinematic diagram, the magnitude of velocity of any arbitrary point is \( \mathbf{v} = \omega \mathbf{r} \), where \( \mathbf{r} \) is the radial distance from the IC to the point. The velocity’s line of action is perpendicular to its associated radial line. Note the velocity has a sense of direction which tends to move the point in a manner consistent with the angular rotation direction.

EXAMPLE

Given: A linkage undergoing motion as shown. The velocity of the block, \( v_D \), is 3 m/s.

Find: The angular velocities of links AB and BD.

Plan: Locate the instantaneous center of zero velocity of link BD.

Solution: Since D moves to the right, it causes link AB to rotate clockwise about point A. The instantaneous center of velocity for BD is located at the intersection of the line segments drawn perpendicular to \( v_B \) and \( v_D \). Note that \( v_B \) is perpendicular to link AB. Therefore we can see that the IC is located along the extension of link AB.
EXAMPLE (continued)

Using these facts,
\[ r_{B/IC} = 0.4 \tan 45^\circ = 0.4 \text{ m} \]
\[ r_{D/IC} = 0.4 / \cos 45^\circ = 0.566 \text{ m} \]

Since the magnitude of \( v_D \) is known, the angular velocity of link BD can be found from \( v_D = \omega_{BD} r_{D/IC} \).

\[ \omega_{BD} = v_D / r_{D/IC} = 3 / 0.566 = 5.3 \text{ rad/s} \]

Link AB is subjected to rotation about A.

\[ \omega_{AB} = v_B / r_{B/A} = (r_{B/IC}) \omega_{BD} / r_{B/A} = 0.4(5.3) / 0.4 = 5.3 \text{ rad/s} \]

EXAMPLE II

Given: The disk rolls without slipping between two moving plates.

\[ v_B = 2v \quad \rightarrow \]
\[ v_A = v \quad \leftarrow \]

Find: The angular velocity of the disk.

Plan: This is an example of the third case discussed in the lecture notes. Locate the IC of the disk using geometry and trigonometry. Then calculate the angular velocity.
Solution:

Using similar triangles:
\[ \frac{x}{v} = \frac{2r-x}{2v} \]

or \[ x = \frac{2}{3}r \]

Therefore \[ \omega = \frac{v}{x} = 1.5\frac{v}{r} \]

GROUP PROBLEM SOLVING

**Given:** The four bar linkage is moving with \( \omega_{CD} \) equal to 6 rad/s CCW.

**Find:** The velocity of point E on link BC and angular velocity of link AB.

**Plan:** This is an example of the second case in the lecture notes. Since the direction of Point B’s velocity must be perpendicular to AB and Point C’s velocity must be perpendicular to CD, the location of the instantaneous center, I, for link BC can be found.
GROUP PROBLEM SOLVING
(continued)

Link AB:

From triangle CBI

\[ IC = 0.346 \text{ m} \]

\[ IB = \frac{0.6}{\sin 60^\circ} = 0.693 \text{ m} \]

\[ v_C = (IC) \omega_{BC} = 3.6 \text{ m/s} \]

\[ \omega_{BC} = v_C/IC = \frac{3.6}{0.346} = 10.39 \text{ rad/s} \]

\[ \omega_{AB} = 6 \text{ rad/s} \]

\[ v_E = (IE) \omega_{BC} \text{ where distance } IE = \sqrt{0.3^2 + 0.346^2} = 0.458 \text{ m} \]

\[ v_E = 0.458(10.39) = 4.76 \text{ m/s} \]

where \( \theta = \tan^{-1}(0.3/0.346) = 40.9^\circ \)
Today’s Objectives:
Students will be able to:
1. Resolve the acceleration of a point on a body into components of translation and rotation.
2. Determine the acceleration of a point on a body by using a relative acceleration analysis.

In-Class Activities:
• Applications
• Translation and Rotation Components of Acceleration
• Relative Acceleration Analysis
• Roll-Without-Slip Motion
• Group Problem Solving

APPLICATIONS
In the mechanism for a window, link AC rotates about a fixed axis through C, while point B slides in a straight track. The components of acceleration of these points can be inferred since their motions are known.

To prevent damage to the window, the accelerations of the links must be limited. How can we determine the accelerations of the links in the mechanism?
APPLICATIONS
(continued)

The forces delivered to the crankshaft, and the angular acceleration of the crankshaft, depend on the speed and acceleration of the piston in an automotive engine.

How can we relate the accelerations of the piston, connection rod, and crankshaft in this engine?

RELATIVE MOTION ANALYSIS: ACCELERATION
(Section 16-7)

The equation relating the accelerations of two points on the body is determined by differentiating the velocity equation with respect to time.

\[
\frac{dv_B}{dt} = \frac{dv_A}{dt} + \frac{dv_{B/A}}{dt}
\]

These are absolute accelerations of points A and B. They are measured from a set of fixed x,y axes.

This term is the acceleration of B with respect to A. It will develop tangential and normal components.

The result is \( a_B = a_A + (a_{B/A})_t + (a_{B/A})_n \)
The relative normal acceleration component \( (a_{B/A})_n \) is \(-\omega^2 r_{B/A}\) and the direction is always from B towards A.

Graphically:

\[ a_B = a_A + (a_{B/A})_t + (a_{B/A})_n \]

The relative tangential acceleration component \( (a_{B/A})_t \) is \( \alpha \times r_{B/A} \) and perpendicular to \( r_{B/A} \).

The relative normal acceleration component \( (a_{B/A})_n \) is \(-\omega^2 r_{B/A}\) and the direction is always from B towards A.

Since the relative acceleration components can be expressed as \( (a_{B/A})_t = \alpha \times r_{B/A} \) and \( (a_{B/A})_n = -\omega^2 r_{B/A} \) the relative acceleration equation becomes

\[ a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A} \]

Note that the last term in the relative acceleration equation is not a cross product. It is the product of a scalar (square of the magnitude of angular velocity, \( \omega^2 \)) and the relative position vector, \( r_{B/A} \).
APPLICATION OF RELATIVE ACCELERATION EQUATION

In applying the relative acceleration equation, the two points used in the analysis (A and B) should generally be selected as points which have a known motion, such as pin connections with other bodies.

In this mechanism, point B is known to travel along a circular path, so $a_B$ can be expressed in terms of its normal and tangential components. Note that point B on link BC will have the same acceleration as point B on link AB.

Point C, connecting link BC and the piston, moves along a straight-line path. Hence, $a_C$ is directed horizontally.

PROCEDURE FOR ANALYSIS

1. Establish a fixed coordinate system.
2. Draw the kinematic diagram of the body.
3. Indicate on it $a_A$, $a_B$, $\omega$, $\alpha$, and $r_{B/A}$. If the points A and B move along curved paths, then their accelerations should be indicated in terms of their tangential and normal components, i.e., $a_A = (a_A)_t + (a_A)_n$ and $a_B = (a_B)_t + (a_B)_n$.
4. Apply the relative acceleration equation:
$$a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A}$$
5. If the solution yields a negative answer for an unknown magnitude, it indicates the sense of direction of the vector is opposite to that shown on the diagram.
**Example**

**Given:** Point A on rod AB has an acceleration of 3 m/s\(^2\) and a velocity of 2 m/s at the instant the rod becomes horizontal.

**Find:** The angular acceleration of the rod at this instant.

**Plan:** Follow the problem solving procedure!

**Solution:** First, we need to find the angular velocity of the rod at this instant. Locating the instant center (IC) for rod AB (which lies above the midpoint of the rod), we can determine \(\omega\):

\[
\omega_A = \frac{v_A}{r_{A/IC}} = \frac{v_A}{(5/\cos 45)} = 0.283 \text{ rad/s}
\]

(continued)

Since points A and B both move along straight-line paths,

\[
a_A = 3 (\cos 45 \, \text{i} - \sin 45 \, \text{j}) \text{ m/s}
\]

\[
a_B = a_B(\cos 45 \, \text{i} + \sin 45 \, \text{j}) \text{ m/s}
\]

Applying the relative acceleration equation

\[
a_B = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A}
\]

\[
(a_B \cos 45 \, \text{i} + a_B \sin 45 \, \text{j}) = (3 \cos 45 \, \text{i} - 3 \sin 45 \, \text{j})
\]

\[
+ (\alpha \, \mathbf{k} \times 10 \, \text{i}) - (0.283)^2(10\text{i})
\]
EXAMPLE (continued)

By comparing the $i, j$ components;

\[ a_B \cos 45 = 3 \cos 45 - (0.283)^2 \] ((10)

\[ a_B \sin 45 = -3 \sin 45 + \alpha \] (10)

Solving:

\[ a_B = 1.87 \text{ m/s}^2 \]

\[ \alpha = 0.344 \text{ rad/s}^2 \]

BODIES IN CONTACT

Consider two bodies in contact with one another without slipping, where the points in contact move along different paths.

In this case, the tangential components of acceleration will be the same, i.e.,

\[ (a_A)_t = (a_{A'})_t \] (which implies \( \alpha_B r_B = \alpha_C r_C \)).

The normal components of acceleration will not be the same.

\[ (a_A)_n \neq (a_{A'})_n \] so \( a_A \neq a_{A'} \).
ROLLING MOTION

Another common type of problem encountered in dynamics involves rolling motion without slip; e.g., a ball or disk rolling along a flat surface without slipping. This problem can be analyzed using relative velocity and acceleration equations.

As the cylinder rolls, point G (center) moves along a straight line, while point A, on the rim of the cylinder, moves along a curved path called a cycloid. If \( \omega \) and \( \alpha \) are known, the relative velocity and acceleration equations can be applied to these points, at the instant A is in contact with the ground.

ROLLING MOTION

(continued)

- **Velocity:** Since no slip occurs, \( v_A = 0 \) when A is in contact with ground. From the kinematic diagram:
  \[
  v_G = v_A + \omega \times r_{G/A} \\
  v_G \hat{i} = \theta + (-\omega \hat{k}) \times (r \hat{j}) \\
  v_G = \omega r \quad \text{or} \quad v_G = \omega \hat{i}
  \]

- **Acceleration:** Since G moves along a straight-line path, \( a_G \) is horizontal. Just before A touches ground, its velocity is directed downward, and just after contact, its velocity is directed upward. Thus, point A accelerates upward as it leaves the ground.
  \[
  a_G = a_A + \alpha \times r_{G/A} - \omega^2 r_{G/A} \implies a_G \hat{i} = a_A \hat{j} + (-\alpha \hat{k}) \times (r \hat{j}) - \omega^2 (r \hat{j})
  \]

Evaluating and equating \( \hat{i} \) and \( \hat{j} \) components:
\[
  a_G = \alpha r \quad \text{and} \quad a_A = \omega^2 r \quad \text{or} \quad a_G = \alpha \hat{i} \quad \text{and} \quad a_A = \omega^2 \hat{j}
\]
Given: The ball rolls without slipping.

Find: The accelerations of points A and B at this instant.

Plan: Follow the solution procedure.

Solution: Since the ball is rolling without slip, $a_O$ is directed to the left with a magnitude of:

$$a_O = \alpha r = (4 \text{ rad/s}^2)(0.5 \text{ ft}) = 2 \text{ ft/s}^2$$

EXAMPLE II (continued)

Now, apply the relative acceleration equation between points O and B.

$$a_B = a_O + \alpha x r_{B/O} - \omega^2 r_{B/O}$$

$$a_B = -2i + (4k) x (0.5i) - (6)^2(0.5i)$$

$$= (-20i + 2j) \text{ ft/s}^2$$

Now do the same for point A.

$$a_A = a_O + \alpha x r_{A/O} - \omega^2 r_{A/O}$$

$$a_A = -2i + (4k) x (0.5i) - (6)^2(0.5j)$$

$$= (-4i - 18j) \text{ ft/s}^2$$
CONCEPT QUIZ

1. If a ball rolls without slipping, select the tangential and normal components of the relative acceleration of point A with respect to G.
   A) \( \alpha x i + \omega^2 r j \)  B) \( -\alpha x i + \omega^2 r j \)
   C) \( \omega^2 r i - \alpha r j \)  D) Zero.

2. What are the tangential and normal components of the relative acceleration of point B with respect to G.
   A) \( -\omega^2 r i - \alpha r j \)  B) \( -\alpha r i + \omega^2 r j \)
   C) \( \omega^2 r i - \alpha r j \)  D) Zero.

GROUP PROBLEM SOLVING

Given: The disk is rotating with \( \omega = 3 \text{ rad/s}, \alpha = 8 \text{ rad/s}^2 \) at this instant.

Find: The acceleration at point B, and the angular velocity and acceleration of link AB.

Plan: Follow the solution procedure.

Solution:
At the instant shown, points A and B are both moving horizontally. Therefore, link AB is translating, meaning \( \omega_{AB} = 0 \).
GROUP PROBLEM SOLVING
(continued)

Draw the kinematic diagram and then apply the relative-acceleration equation:

\[ a_B = a_A + \alpha_{AB} \times r_{B/A} - \omega^2_{AB} r_{B/A} \]

Where

\[ a_{An} = 0.2 \times 3^2 = 1.8 \text{ m/s}^2 \]
\[ a_{At} = 0.2 \times 8 = 1.6 \text{ m/s}^2 \]
\[ r_{B/A} = 0.4 \cos 30^\circ i - 0.4 \sin 30^\circ j \]
\[ \alpha_{AB} = \alpha_{AB} k, \omega_{AB} = 0 \]

\[ a_B = (1.6 i - 1.8 j) + \alpha_{AB} k \times (0.4 \cos 30^\circ i - 0.4 \sin 30^\circ j) \]
\[ = (1.6 + 0.4 \sin 30^\circ \alpha_{AB}) i + (-1.8 + 0.4 \cos 30^\circ \alpha_{AB}) j \]

GROUP PROBLEM SOLVING
(continued)

By comparing \( i, j \) components:

\[ a_B = 1.6 + 0.4 \sin 30^\circ \alpha_{AB} \]
\[ 0 = -1.8 + 0.4 \cos 30^\circ \alpha_{AB} \]

Solving:

\[ a_B = 2.64 \text{ m/s}^2 \]
\[ \alpha_{AB} = 5.20 \text{ rad/s}^2 \]