HW#3 12-107, 117, 133, 141, 12-152, 163

Problem 12–107

The car travels along the curve having a radius of \( R \). If its speed is uniformly increased from \( v_1 \) to \( v_2 \) in time \( t \), determine the magnitude of its acceleration at the instant its speed is \( v_3 \).

Given:

\[
v_1 = 15 \, \text{m/s} \quad t = 3 \, \text{s}
\]

\[
v_2 = 27 \, \text{m/s} \quad R = 300 \, \text{m}
\]

\[
v_3 = 20 \, \text{m/s}
\]

Solution:

\[
a_t = \frac{v_2 - v_1}{t} \quad a_n = \frac{v_3^2}{R} \quad a = \sqrt{a_t^2 + a_n^2} \quad a = 4.22 \, \text{m/s}^2
\]
Problem 12–117

Cars move around the “traffic circle” which is in the shape of an ellipse. If the speed limit is posted at \( v \), determine the maximum acceleration experienced by the passengers.

Given:

\[
\begin{align*}
v & = 60 \text{ km/hr} \\
a & = 60 \text{ m} \\
b & = 40 \text{ m}
\end{align*}
\]

Solution:

Maximum acceleration occurs where the radius of curvature is the smallest. In this case that happens when \( y = 0 \).

\[
\begin{align*}
x(y) &= a \sqrt{1 - \left(\frac{y}{b}\right)^2} \\
x'(y) &= \frac{d}{dy} x(y) \\
x''(y) &= \frac{d}{dy} x'(y) \\
ρ(y) &= \frac{1 + x'(y)^2}{x''(y)} \\
ρ_{\min} &= ρ(0m) \\
ρ_{\min} &= 26.667 \text{ m} \\
a_{\max} &= \frac{v^2}{ρ_{\min}} \\
a_{\max} &= 10.42 \text{ m/s}^2
\end{align*}
\]
Problem 12-133

The truck travels at speed \( v_0 \) along a circular road that has radius \( \rho \). For a short distance from \( s = 0 \), its speed is then increased by \( \frac{dv}{dr} = bs \). Determine its speed and the magnitude of its acceleration when it has moved a distance \( s_I \).

Given:

\[
v_0 = 4 \text{ m/s} \]
\[
\rho = 50 \text{ m} 
\]
\[
b = \frac{0.05}{s^2} 
\]
\[
s_I = 10 \text{ m} 
\]

Solution:

\[
a_t = v \left( \frac{d}{ds} v \right) = bs 
\]
\[
\int_{v_0}^{v_I} v \, dv = \int_0^{s_I} b \, s \, ds 
\]
\[
\frac{v_I^2}{2} - \frac{v_0^2}{2} = \frac{bs_I^2}{2} 
\]

\[
v_I = \sqrt{v_0^2 + b s_I^2} 
\]
\[
v_I = 4.58 \text{ m/s} 
\]

\[
a_t = bs_I 
\]
\[
a_n = \frac{v_I^2}{\rho} 
\]
\[
a = \sqrt{a_t^2 + a_n^2} 
\]
\[
a = 0.653 \text{ m/s^2} 
\]
Problem 12-141

If a particle’s position is described by the polar coordinates \( r = a \sin b \theta \) and \( \theta = ct \), determine the radial and tangential components of its velocity and acceleration when \( t = t_f \).

Given: \( a = 2 \text{ m} \) \hspace{1cm} \( b = 2 \text{ rad} \) \hspace{1cm} \( c = 4 \frac{\text{rad}}{\text{s}} \) \hspace{1cm} \( t_f = 1 \text{ s} \)

Solution: \( t = t_f \)

\[
\begin{align*}
    r &= (a) \sin(b \cdot c \cdot t) \\
    r' &= a b c \cos(b \cdot c \cdot t) \\
    r'' &= -a b^2 c^2 \sin(b \cdot c \cdot t) \\
    \theta &= c t \\
    \theta' &= c \\
    \theta'' &= 0 \frac{\text{rad}}{\text{s}^2}
\end{align*}
\]

\[
\begin{align*}
    v_r &= r' \\
    v_r &= 2.328 \frac{\text{m}}{\text{s}}
\end{align*}
\]

\[
\begin{align*}
    v_\theta &= r \theta' \\
    v_\theta &= 7.915 \frac{\text{m}}{\text{s}}
\end{align*}
\]

\[
\begin{align*}
    a_r &= r'' - r \theta'^2 \\
    a_r &= -158.3 \frac{\text{m}}{\text{s}^2}
\end{align*}
\]

\[
\begin{align*}
    a_\theta &= r \theta' + 2r' \theta' \\
    a_\theta &= -18.624 \frac{\text{m}}{\text{s}^2}
\end{align*}
\]
Problem 12-152

At the instant shown, the watersprinkler is rotating with an angular speed $\theta$ and an angular acceleration $\theta'$. If the nozzle lies in the vertical plane and water is flowing through it at a constant rate $r'$, determine the magnitudes of the velocity and acceleration of a water particle as it exits the open end, $r$.

Given:

$\theta = 2 \text{ rad/s}$ \hspace{1cm} $\theta' = 3 \text{ rad/s}^2$

$r' = 3 \text{ m/s}$ \hspace{1cm} $r = 0.2 \text{ m}$

Solution:

$v = \sqrt{r^2 + (r\theta)^2}$ \hspace{1cm} $v = 3.027 \text{ m/s}$

$a = \sqrt{(r\theta')^2 + (r\theta'' + 2r'\theta)^2}$ \hspace{1cm} $a = 12.625 \text{ m/s}^2$
Problem 12–163

For a short time the bucket of the backhoe traces the path of the cardioid \( r = a(1 - \cos \theta) \). Determine the magnitudes of the velocity and acceleration of the bucket at \( \theta = \theta_1 \) if the boom is rotating with an angular velocity \( \theta' \) and an angular acceleration \( \theta'' \) at the instant shown.

Given:
\[
\begin{align*}
    a &= 25 \text{ ft} \quad \theta' = 2 \frac{\text{rad}}{\text{s}} \\
    \theta_1 &= 120 \text{ deg} \quad \theta'' = 0.2 \frac{\text{rad}}{\text{s}^2}
\end{align*}
\]

Solution:

\[
\begin{align*}
    \theta &= \theta_1 \\
    r &= a(1 - \cos(\theta)) \quad r' = a \sin(\theta) \theta' \\
    r'' &= a \sin(\theta) \theta'' + a \cos(\theta) \theta'^2 \\
    v &= \sqrt{r'^2 + (r \theta')^2} \quad \text{\( v = 86.6 \frac{\text{ft}}{\text{s}} \)} \\
    a &= \sqrt{(r'' - r \theta'^2)^2 + (r \theta' + 2r' \theta)^2} \quad \text{\( a = 266 \frac{\text{ft}}{\text{s}^2} \)}
\end{align*}
\]