5.1-1 The bandwidths of \( f_1(t) \) and \( f_2(t) \) are 100 kHz and 150 kHz, respectively. Therefore the Nyquist sampling rate for \( f_1(t) \) is 200 kHz and for \( f_2(t) \) is 300 kHz.

Also \( f_1^2(t) \leftrightarrow \frac{1}{2} F_1(\omega) \ast F_1(\omega) \) and from the width property of convolution the bandwidth of \( f_1^2(t) \) is twice the bandwidth of \( f_1(t) \) and that of \( f_2^2(t) \) is three times the bandwidth of \( f_2(t) \) (see also Prob. 4.3-10). Similarly the bandwidth of \( f_1(t) \cd f_2(t) \) is the sum of the bandwidth of \( f_1(t) \) and \( f_2(t) \). Therefore the Nyquist rate for \( f_1^2(t) \) is 400 kHz. for \( f_2^2(t) \) is 900 kHz. for \( f_1(t) \cd f_2(t) \) is 500 kHz.

5.1-2

(a) \[
\text{sinc}^2(100\pi t) \iff 0.01\Delta\left(\frac{200\pi}{240\pi}\right)
\]

The bandwidth of this signal is 200 \( \pi \) rad/s or 100 Hz. The Nyquist rate is 200 Hz (samples/sec).

(b) The Nyquist rate is 200 Hz, the same as in (a), because multiplication of a signal by a constant does not change its bandwidth.

(c) \[
\text{sinc}(100\pi t) + 3\text{sinc}^2(60\pi t) \iff 0.01\text{rect}\left(\frac{200\pi}{240\pi}\right) + \frac{1}{20}\Delta\left(\frac{240\pi}{240\pi}\right)
\]

The bandwidth of \( \text{rect}\left(\frac{200\pi}{240\pi}\right) \) is 50 Hz and that of \( \Delta\left(\frac{240\pi}{240\pi}\right) \) is 60 Hz. The bandwidth of the sum is the higher of the two, that is, 60 Hz. The Nyquist sampling rate is 120 Hz.

(d) \[
\text{sinc}(50\pi t) \iff 0.02\text{rect}\left(\frac{100\pi}{200\pi}\right)
\]
\[
\text{sinc}(100\pi t) \iff 0.01\text{rect}\left(\frac{200\pi}{200\pi}\right)
\]

The two signals have bandwidths 25 Hz and 50 Hz respectively. The spectrum of the product of two signals is 1/2\( \pi \) times the convolution of their spectra. From width property of the convolution, the width of the convoluted signal is the sum of the widths of the signals convolved. Therefore, the bandwidth of \( \text{sinc}(50\pi t)\text{sinc}(100\pi t) \) is \( 25 + 50 = 75 \) Hz. The Nyquist rate is 150 Hz.

5.1-4

(a) When the input to this filter is \( \delta(t) \), the output of the summer is \( \delta(t) - \delta(t - T) \). This acts as the input to the integrator. And, \( h(t) \), the output of the integrator is:

\[
h(t) = \int_0^t [\delta(\tau) - \delta(\tau - T)] d\tau = u(t) - u(t - T) = \text{rect}\left(\frac{t - \frac{T}{2}}{\frac{T}{2}}\right)
\]

The impulse response \( h(t) \) is shown in Fig. S5.1-4a.

(b) The transfer function of this circuit is:

\[
H(\omega) = T\text{sinc}\left(\frac{\omega T}{2}\right)e^{-j\omega T/2}
\]

and

\[
|H(\omega)| = T\left|\text{sinc}\left(\frac{\omega T}{2}\right)\right|
\]

The amplitude response of the filter is shown in Fig. S5.1-4b. Observe that the filter is a lowpass filter of bandwidth \( 2\pi/T \) rad/s or \( 1/T \) Hz.

The impulse response of the circuit is a rectangular pulse. When a sampled signal is applied at the input, each sample generates a rectangular pulse at the output, proportional to the corresponding sample value. Hence the output is a staircase approximation of the input as shown in Fig. S5.1-4c.
5.1-8
(a) The bandwidth is 15 kHz. The Nyquist rate is 30 kHz.
(b) $65536 = 2^{16}$, so that 16 binary digits are needed to encode each sample.
(c) $30000 \times 16 = 480000 \text{ bits/s}$.
(d) $44100 \times 16 = 705600 \text{ bits/s}$.

5.2-3

\[ f(t) = e^{-t}u(t) \quad \quad \quad F(\omega) = \frac{1}{j\omega + 1} \]

(a) We take the folding frequency $F_s$ to be the frequency where $|F(\omega)|$ is 1% of its peak value, which happens to be 1 (at $\omega = 0$). Hence,

\[ |F(\omega)| \approx \frac{1}{\omega} = 0.01 \Rightarrow \omega = 2\pi B = 100 \]

This yields $B = 50/\pi$, and $T \leq 1/2B = \pi/100$. Let us round $T$ to 0.03125, resulting in 32 samples per second. The time constant of $e^{-t}$ is 1. For $T_0$, a reasonable choice is 5 to 6 time constants or more. Value of $T_0 = 5$ or 6 results in $N_0 = 160$ or 192, neither of which is a power of 2. Hence, we choose $T_0 = 8$, resulting in $N_0 = 32 \times 8 = 256$, which is a power of 2.

(b)\[ |F(\omega)| = \frac{1}{\sqrt{\omega^2 + 1}} \approx \frac{1}{\omega} \quad \omega \gg 1 \]

We take the folding frequency $F_s$ to be the 99% energy frequency as explained in Example 4.16. From the results in Example 4.16, (with $a = 1$) we have

\[ \frac{0.99\pi}{2} = \tan^{-1} \frac{W}{a} \Rightarrow W = 63.66a = 63.66 \text{ rad/sec.} \]

This yields $B = \frac{W}{2\pi} = 10.13 \text{ Hz}$. Also $T \leq 1/2B = 0.04936$. This results in the sampling rate $\frac{1}{2} = 20.26 \text{ Hz}$. Also $T_0 = 8$ as explained in part (a). This yields $N_0 = 20.26 \times 8 = 162.08$, which is not a power of 2. Hence, we choose the next higher value, that is $N_0 = 256$, which yields $T = 0.03125$ and $T_0 = 8$, the same as in part (a).

5.2-4

\[ f(t) = \frac{e^{-t}}{t^2 + 1} \]

Application of duality property to pair 3 (Table 4.1) yields

\[ \frac{2}{t^2 + 1} \iff 2\pi e^{-|\omega|} \]

Following the approach of Prob. 5.2-2, we find that the peak value of $|F(\omega)| = 2\pi e^{-|\omega|}$ is $2\pi$ (occurring at $\omega = 0$). Also, $2\pi e^{-|\omega|}$ becomes $0.01 \times 2\pi$ (1% of the peak value) at $\omega = \ln 100 = 4.605$. Hence, $D = 4.605/2\pi = 0.735 \text{ Hz}$, and $T \leq 1/2B = 0.882$. Also,

\[ f(0) = 2 \quad \text{and} \quad f(t) \approx \frac{2}{t^2} \quad t \gg 1 \]

Choose $T_0$ (the duration of $f(t)$) to be the instant where $f(t)$ is 1% of $f(0)$.

\[ f(T_0) = \frac{2}{T_0^2 + 1} = \frac{2}{100} \Rightarrow T_0 \approx 10 \]

This results in $N_0 = T_0/T = 10/0.682 = 14.56$. We choose $N_0 = 16$, which is a power of 2. This yields $T = 0.625$ and $T_0 = 10$.

(b) The energy of this signal is

\[ E_f = \frac{2}{2\pi} \int_{0}^{\infty} (2\pi)^2 e^{-2\omega} d\omega = 2\pi \]

The energy within the band from $\omega = 0$ to $W$ is given by

\[ E_{W} = \frac{8\pi^2}{2\pi} \int_{0}^{W} e^{-2\omega} d\omega = 2\pi(1 - e^{-2W}) \]
But $E_W = 0.99 E_f = 0.99 \times 2\pi$. Hence,

$$0.99(2\pi) = 2\pi(1 - e^{-2W}) \Rightarrow W = 2.303$$

Hence, $B = W/2\pi = 0.366$ Hz. Thus, $T \leq 1/2B = 1.366$ Hz. Also, $T_0 = 10$ as found in part (a). Hence, $N_0 = T_0/T = 7.32$. We select $N_0 = 8$ (a power of 2), resulting in $N_0 = 8$ and $T = 1.25$.

![Diagram](image)

Figure S5.2-5