Problem 1
1) Find the mask for the horizontal and vertical edge detection filters that approximate the derivative using

\[ f'(x) = \frac{-f(x + 2h) + 8f(x + h) - 8f(x - h) + f(x - 2h)}{12h} \]  

We take \( h = 1 \), no need to calculate the magnitude.

Problem 2
Apply the following morphological operations
- dilation
- erosion
- opening
- closing
to the image given by:

\[
C = \begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1
\end{bmatrix}
\]

(2)

Use structuring elements

\[
B_1 = \begin{bmatrix}
1 & 0 \\
1 & 1
\end{bmatrix}
\]

(3)

and

\[
B_2 = \begin{bmatrix}
1 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]

(4)

Problem 3
Consider the binary image represented by matrix \( A \):

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(5)

- What kind of operation and which structuring element will you apply to remove the horizontal lines?
- What kind of operation and which structuring element will you apply to remove the vertical lines?
Keep the structuring elements as simple as possible.

Problem 4
We define the following structuring elements:

\[
B_1 = \begin{bmatrix}
1 & 1 \\
0 & 0
\end{bmatrix}
\]

(6)

\[
B_2 = \begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

(7)

\[
B_3 = \begin{bmatrix}
1 & 1 \\
1 & 1
\end{bmatrix}
\]

(8)
\[ B_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \]  

and image A:

\[ A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]  

Find the erosion and dilation of A with each of the masks above.

Problem 5

1) A majority filter is a
   - special median filter
   - special convolution filter
   - generalization of the Laplacian filter
   - generalization of the median filter
   - generalization of the mean filter

2) The circularity ratio \( R_c \) is a measure of compactness defined as:
   \[ R_c = \frac{4\pi A}{P^2} \]  
   where \( A \) is the area and \( P \) is the perimeter. The most circular object is
   - polygon with 12 sides
   - polygon with 3 sides
   - straight line
   - elephant

Problem 6

For the following image

\[ C = \begin{bmatrix} 123 & 12 & 43 \\ 12 & 4 & 3 \\ 44 & 55 & 240 \end{bmatrix} \]  

1) Perform filtering with rank 3 filter
2) Perform filtering with max filter
3) Perform filtering with a median filter.

Problem 7

We want to perform hit and miss operation to find specific shapes. The original image and the structuring elements are shown in figure 1 and 2. We want to do the following
   - Isolate the shape given by \( T_1 \)
   - Replace it by the shape given by \( C \)

We want to perform morphological operation to achieve these goals.
   1) Perform erosion with \( T_1 \)
   2) Perform hit and miss with \( T_1 \) and \( T_2 \)
   3) How to replace the shape given by \( T_1 \) by the shape given by \( C \)?
Fig. 1. Image and structuring element
Fig. 2. Image and structuring element

Original image

| 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 |
| 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 |
| 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 |
| 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 |
| 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 |
| 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 |
| 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 |
| 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 |
| 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1 1 0 0 0 |
| 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 |
| 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 |
| 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 |
| 0 0 0 0 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 |
| 0 0 0 0 0 1 1 1 0 0 0 0 0 1 1 1 0 0 0 |
| 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 |

Structuring elements

$C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$T_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ \end{bmatrix}$

$T_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Fig. 2. Image and structuring element