Problem 1

The path of the robot is shown in figure 1. Our goal is to determine the location of the robot after 10 seconds using different methods:

- integration by hand
- integration using a symbolic toolbox
- using odometry equation (numerical integration)

We have the following information:

\[ v = 3 \text{ m/s}, \quad \omega = 0.5 \text{rad/s}, \quad ICC = 6 \text{m} \]
\[ \theta_0 = \pi/2, \quad x_0 = 6 \text{m}, \quad y_0 = 0 \text{m} \]  \hspace{1cm} (1)

Recall that when both \( \omega \) and \( v \) are constant:

\[ x(t) = \int_0^t v \cos(\omega t + \theta_0) + x_0 \]  \hspace{1cm} (2)
\[ y(t) = \int_0^t v \sin(\omega t + \theta_0) + y_0 \]  \hspace{1cm} (3)
\[ \theta(t) = \omega t + \theta_0 \]  \hspace{1cm} (4)

1) Perform integration by hand to determine the position and the orientation of the robot after 10 seconds.
2) Perform integration using a symbolic toolbox to determine the position and the orientation of the robot after 10 seconds. Hint: to find

\[ \int_0^1 \sin(t) dt \]  \hspace{1cm} (5)

you can use

\[ \text{syms t} \]  \hspace{1cm} (6)
\[ \text{int(sin(t),0,1)} \]  \hspace{1cm} (7)
3) Write code to implement the odometry equations to determine the position and orientation of the robot after 2 seconds. Take a sampling time of $T = 0.01$ s. Recall that the odometry equations can be written as follows:

$$x = x + T * v * \cos(\theta)$$  \hspace{1cm} (8) \\
$$y = y + T * v * \sin(\theta)$$  \hspace{1cm} (9) \\
$$\theta = T * w + \theta$$  \hspace{1cm} (10)

4) Plot the robot's path in the time interval $[0, 10s]$ 
5) Are your results consistent?

Problem 2

A rotation matrix $R$ is generated by the following rotations:

- $60^\circ$ about current $z_0$
- $30^\circ$ about current $y_0$
- $90^\circ$ about current $x_0$

Find matrix $R$.

Problem 3

A rotation matrix $R$ is generated by the following rotation:

- $90^\circ$ about current $z_0$

Answer the following questions:

1) Without any calculations, what is the equivalent axis ($\vec{k}$) and its corresponding angle.
2) Use the equations to obtain the equivalent axis ($\vec{k}$) and its corresponding angle. Does it confirm the result of the previous question?

Problem 4

Suppose frames $o_0x_0y_0z_0$ and $o_1x_1y_1z_1$ are related by rotation matrix:

$$R^0_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$  \hspace{1cm} (11)

1) Find $R^0_1$. You can use Matlab or other numerical tools.
2) Find the rotation axis $\vec{k}$ and the rotation angle $\theta$ corresponding to $R^0_1$. 