PROBLEM 1: CALIBRATION OF SHARP IR SENSOR

In this problem we want to determine the calibration curve of the sharp sensor shown in figures 1 and 2. It is clear that the relationship between the input (distance) and output (voltage) is nonlinear. In order to have one to one relationship, we ignore any distance that is less than $3\text{cm}$. The data points are shown in table 3. To obtain the calibration curve, we propose to use polynomial regression model as follows

$$d_i = a_0 + a_1 v_i + a_2 v_i^2 + \ldots + a_k v_i^k$$ \hspace{1cm} (1)

$$i = 1, \ldots, n$$ \hspace{1cm} (2)

where

- $d_i$ is the distance in centimeters and $v_i$ is the voltage in volts.
- $a_0, a_1, \ldots, a_k$ are the coefficients of the polynomial, to be determined.
- $k$ is the degree of the polynomial
- $n$ is the number of data points.

For $n$ measurements, system (1) can be written under matrix form as follows

$$
\begin{bmatrix}
    d_0 \\
    d_1 \\
    \vdots \\
    d_n
\end{bmatrix} = 
\begin{bmatrix}
    1 & v_1 & \cdots & v_1^k \\
    1 & v_2 & \cdots & v_2^k \\
    \vdots & \vdots & \ddots & \vdots \\
    1 & v_n & \cdots & v_n^k
\end{bmatrix}
\begin{bmatrix}
    a_0 \\
    a_1 \\
    \vdots \\
    a_n
\end{bmatrix}
$$ \hspace{1cm} (3)

System (3) can be written under matrix form as follows

$$D = VA$$ \hspace{1cm} (4)

where

$$D = [d_0 \ d_1 \ \cdots \ d_n]^T$$ \hspace{1cm} (5)

$$V = 
\begin{bmatrix}
    1 & v_1 & \cdots & v_1^k \\
    1 & v_2 & \cdots & v_2^k \\
    \vdots & \vdots & \ddots & \vdots \\
    1 & v_n & \cdots & v_n^k
\end{bmatrix}$$ \hspace{1cm} (6)

and

$$A = [a_0 \ a_1 \ \cdots \ a_n]^T$$ \hspace{1cm} (7)

The least squares method can be used to obtain the coefficient of the polynomials as follows

$$A = (V^T V)^{-1} V^T D$$ \hspace{1cm} (8)

This equation is called the modified equation inverse.

1) Use Matlab, Python or similar tools to determine the coefficients when a polynomial of degree six ($k = 6$) is used.
2) Use Matlab, Python or similar tools to determine the coefficients when a polynomial of degree four ($k = 4$) is used.
3) Plot the polynomials on the same graph as the data points shown in figure 2. (These are the points shown in table 3.)
4) Use your polynomials to compute the absolute error for the points shown in table 4 and complete the table.
5) From the polynomial of degree six, what is the distance if the voltage reads 0.85\textit{V}?
Table 3: Data for Sharp sensor.

<table>
<thead>
<tr>
<th>The distance in centimeters</th>
<th>The voltage in volts</th>
<th>Error in distance using 4th degree polynomial</th>
<th>Error in distance using 6th degree polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0000</td>
<td>1.2300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.0000</td>
<td>0.6500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25.0000</td>
<td>0.5500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.0000</td>
<td>0.4400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35.0000</td>
<td>0.3750</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Specific calibration values for comparison