Problem 1

Figure 1–top illustrates a temperature control system with a proportional controller. The desired value for the temperature is 25°C. The open loop transfer function is given by

\[ G(s) = \frac{1}{s^2 + as + 1} \]  (1)

1) Write an equation for \( R(s) \).
2) Find the closed loop transfer function
3) Use the Routh-Hurwitz test to determine the conditions on the parameter \( a \) and the gain \( K \) for the system to be stable. For the rest of the problem we take \( a = 10 \).
4) What is the type of the system?
5) Determine the steady state error as a function of \( K \).
6) Determine the gain \( K \) so that the steady error for the desired input temperature is never more than 1°C.
7) Write the closed loop system in state space.
8) For \( K = 20 \), determine the steady state error using the state space model.
9) Figure 2 shows the system response when the desired temperature is 25°C, what is the steady state error? Deduce the numerical value of the gain.
10) What is the settling time and the percent overshoot?
11) Determine the Braditivity of the steady state error with respect to the gain and to parameter \( a \).

Problem 2

This problem is a continuation of the previous one where an integral controller is added to improve the steady state error. The system’s closed loop is shown in figure 1–bottom. The desired value for the temperature is 25°C.

1) Find the closed loop transfer function
2) Use the Routh-Hurwitz test to determine the conditions on the parameter \( a \) and the gain \( K \) for the system to be stable. For the rest of the problem we take \( a = 10 \).
3) Determine the steady state error as a function of $K$.
4) Determine the gain $K$ so that the steady error for the desired input is never more than $1^\circ C$.
5) Figure 3 shows the system response when the desired temperature is $25^\circ$, what is the steady state error?
6) What is the settling time and the percent overshoot?