SHAPE DESCRIPTORS

Objects in an image are a collection of pixels. In order to describe an object or distinguish between objects, we need to understand the properties and arrangement of the group of pixels forming the object. Shape descriptors are sets of numbers that can be used to describe a given shape or object. Thus, good descriptors make object recognition easier. Descriptors should have three important properties:

- Completeness: two objects should have the same descriptors if and only if they have the same shape
- Uniqueness: an object of a given shape should have a unique representation under a given descriptor.
- Invariance: descriptors should be invariant to geometric transformations such as rotation, translation, and scaling.

Is it possible to reconstruct the shape from its descriptors? The answer is no in general.

SIMPLE DESCRIPTORS

For a given shape $S$, we can define the following descriptors:

1) Area $A(S)$: number of the pixels in an object.

$$A(S) = \sum_{i,j} f(i,j) \quad (1)$$

For a binary image, $A$ is just the number of pixels in the object.

2) Perimeter $P(S)$: number of pixels in the boundary of the object. It is clear that the area and perimeter are not invariant to scaling.

3) Compactness $C(S)$: it is defined as the ratio of the perimeter squared to the area. This measure is dimensionless and is independent of scaling:

$$C = \frac{\text{perimeter}^2}{\text{area}} \quad (2)$$

4) Circularity ratio $R_c$: slightly different measure from compactness, it is defined as:

$$R_c = \frac{4\pi A(S)}{P^2(S)} \quad (3)$$

The circle is the most circular shape.

Example

Calculate the compactness of the circle and a square.

a) Circle: compactness=4$\pi$.

b) Square: compactness =16.

c) The value of $R_c$ is 1 for a circle and $\pi/4$ for a square.

5) Irregularity is another descriptor that can be used to characterize shapes. Irregularity is defined as follows

$$I(S) = \frac{\pi \max ((i - \bar{i})^2 + (j - \bar{j})^2)}{A(S)} \quad (4)$$

where $(i,j)$ represent the coordinates of the center of mass. What is the irregularity of a circle? Answer: 1.

6) Convex area: this is another definition of the area. The area obtained by stretching a rubber band around the object.

$S = \frac{\text{object area}}{\text{convex area}} \quad (5)$

7) Solidity: solidity is the ratio of the object area to the convex area.

8) Euler number $E$: this is a topological measure. It is given by

$$E = C - H \quad (6)$$

where $H$ is the number of holes and $C$ is the number of connected (continuous) elements. For 3, B and 9, the Euler number is 1, $-1$ and 0, respectively.

9) Eccentricity is the measure of aspect ratio, that is the ratio of length of major axis to minor axis.

10) Elongation: the ratio of the height and width of a rotated minimal bounding box. In other words, rotate a rectangle so that it is the smallest rectangle in which the shape fits. Then compare its height to its width.

11) Chain codes: It is a representation of the boundary, where the boundary of the shape is translated to a sequence of numbers. It is possible to use 4 or 8 dimensional chain codes. Chain codes depend on the starting point. We need to make it invariant to the starting point. One way is by shifting the sequence so that the smallest integer is the starting point. An illustration of chain codes in shown in figure 1.

12) Profiles: Profiles are projections of the shape to x-axis and y-axis, we obtain two 1D functions. The profiles are given by

$$Pro_x(i) = \sum_j f(i,j)$$

$$Pro_y(j) = \sum_i f(i,j) \quad (7)$$

Profiles are simply the sum of the pixels in each row and column. In order to illustrates image profiles, consider image A below
Fig. 1. Illustration of chain code

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0
\end{bmatrix}
\]

The profiles are

\[
\text{Prof}_y = [3 \ 3 \ 3 \ 3 \ 3 \ 8 \ 8 \ 2 \ 2 \ 2],
\]
\[
\text{Prof}_x = [2 \ 2 \ 2 \ 2 \ 2 \ 10 \ 10 \ 7 \ 0 \ 0].
\]

(8)

13) Rectangularity: Represents how rectangular a shape is. It can be calculated using

\[
\text{rectangularity} = \frac{A_s}{A_r}
\]

(9)

where

- \( A_s \): area of the shape
- \( A_r \): area of the minimum boundary rectangle

The rectangularity of a circle is given by

\[
\text{rectangularity} = \frac{\pi R^2}{4R^2} = \frac{\pi}{4}
\]

(10)

Shape signature

Shape signatures use a combination of descriptors. There are several possibilities. One popular solution is to use the angles and distances of the shape pixels to a particular point such the centroid for example. The letter C is figure 2-(a) has the following shape signature:

Distance: \( \{ \sqrt{2}, 1, \sqrt{2}, 1, \sqrt{2}, 1, \sqrt{2} \} \)

(11)

and

Angle: \( \{ 45, 90, 135, 180, 225, -90, -45 \} \)

(12)

A. Example

Find the shape signature \((\theta, r)\) for a circle, a rectangle and a square.

The shape signatures \((\theta, r)\) for a circle, a rectangle and a square are shown in figure 4.

Regionprops

Matlab has a built-in function called \textit{regionprops} that supports many descriptors such as

- 'Area'
- 'EulerNumber'
- 'Perimeter'
- 'Eccentricity'
• ‘Centroid’
• ‘ConvexArea’

**Example**

For the letter A image shown in figure 6, we have the following descriptors:

- \( s = \text{regionprops}(o, \text{‘area’}) \)
- \( s = \text{Area}: 1652 \)
- \( s = \text{regionprops}(o, \text{‘centroid’}) \)
- \( s = \text{Centroid}: [60.9655, 63.9885] \)
- \( s = \text{regionprops}(o, \text{‘ConvexArea’}) \)
- \( s = \text{ConvexArea}: 5224 \)
- \( s = \text{regionprops}(o, \text{‘Solidity’}) \)
- \( s = \text{Solidity}: 0.3162 \)
- \( s = \text{regionprops}(o, \text{‘Eccentricity’}) \)
- \( s = \text{Eccentricity}: 0.4433 \)
- \( s = \text{regionprops}(o, \text{‘Perimeter’}) \)
- \( s = \text{Perimeter}: 440.9777 \)

These descriptors are obtained using the Matlab command `regionprops`.

**B. Fourier descriptors**

Fourier descriptors cover the "statistics of shapes" for a binary image. They are used to describe shapes based on the DFT. There are different ways to obtain the Fourier descriptors. One common approach is to use the boundary line of a two-dimensional object, where the boundary is approximated by some one-dimensional function \( s(k) \). A simple way is to convert the coordinates \((x(k), y(k))\) of the boundary points \( k = 0, \ldots, N \) to complex numbers:

\[
s(k) = x(k) + jy(k) \tag{13}
\]

The DFT of \( s(k) \) is given by

\[
a(u) = \sum_{k=0}^{N-1} s(k) e^{- \frac{2\pi j ku}{N}} \tag{14}
\]

The complex coefficients \( a \) are called the Fourier descriptors of the boundary.

**C. Polynomial approximation**

A digital boundary can be treated as a 1D function and approximated by a polynomial such as

\[
y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n \tag{15}
\]

The coefficients are the shape descriptors in this case. Traditional function approximation methods such as least squares and spline methods can be used in this case.

**D. Moments**

Statistical moments can be used to describe the arrangement of the pixels in an image. Mechanical moments describe the rate of change of the momentum and statistical moments
describe the rate of change in the shape’s area. The two dimensional Cartesian moment of order \( p - q \) is given by

\[
m_{pq} = \sum_{i,j} i^p j^q f(i,j)
\]  

(16)

Zero order moment is given by

\[
m_{00} = \sum_{i,j} f(i,j)
\]  

(17)

In a binary image, this quantity represent the number of pixels in the image. In the case of a binary image the zero order moment is simply the number of pixels in the shape. First