Robotic system components

A robotic system has three major components:

- Actuators: the muscles of the robot
- Sensors: provide information about the environment and also about the internal state of the robot.
- Controller: the brain of the robot.

Robotic manipulators

A robotic manipulator is composed of links connected by joints, together they form a kinematic chain. Manipulators are also called robotic arms. Applications of robotic manipulators include pick and place, welding, painting, etc. Robotic surgery is among the most recent application of robotic manipulators.

Some definitions:

- Configuration of the manipulator: The configuration of the manipulator is the location and orientation of every point in the manipulator. If we know the joint variables and the links' length, we can determine the configuration. To each joint, we associate a variable that we call the joint variable. Therefore, the configuration is represented by a vector of the joint variables and will be denoted by
  \[ q = [q_1, \ldots, q_n]^T. \] (1)
  where \( q_i \) is the variable associated with joint \( i \). For a manipulator of \( n \) joints, the dimension of the configuration space is simply \( n \).
- Degree of freedom (DOF): We need three coordinates to specify the location of a point. For a rigid body, we need three orientations in addition to the three coordinates. Thus for a rigid body we have a total of six degrees of freedom. A degree of freedom refers to the ability to move in a single independent direction. For a robotic manipulator, the degree of freedom is the minimum number of parameters specifying the configuration space. The number of joints determines the DOF. Typically, for each degree of freedom, a joint is required. You need to have at least six degrees of freedom if you want the manipulator to reach any point in its workspace. A manipulator that has more than six degrees is called kinematically redundant. A snake robot is often called hyper redundant because it has many more than six degrees of freedom.
- Configuration space: The configuration space is the set of all possible configurations.
- Workspace: It is the total area swept out by the end effector as the manipulator executes all possible motions. To be more specific, the workspace includes the reachable and dexterous workspace.
- End effector: the tool located at the end of the manipulator is called end effector. Examples of end effectors include grippers and welding guns.
- Reachable workspace: The set of point reachable by the manipulator.
- Dexterous workspace: The set of points the manipulator can reach with an arbitrary orientation of the end effector.
- Parallel Manipulators: A subset of links that form a closed chain, which results in better structural stability and more accuracy.
- State space: The configuration space deals with the geometry, the state space is more general and deals with the manipulators dynamics.

The dexterous workspace is a subset of the reachable workspace.

Types of joints

There exist different types of joints, the most popular are:

- rotary or revolute
- linear or prismatic
- spherical

To each joint we associate a configuration variable. In the case of a rotary joint, the joint variable is an angle and in the case of a prismatic joint, the joint variable is a displacement.

Common kinematic arrangements

Only the first three joints are used to classify robotic manipulators. The most common arrangements are the following:

- Articulated RRR (also called revolute)
- Spherical RRP
- Scara RRP
- Cylindrical RPP
- Cartesian PPP

Figures 7 and 8 show the common arrangements and their corresponding workspace.

Rigid body motion and homogeneous transformation

Consider a manipulator performing pick and place tasks, the manipulator needs to have accurate estimation of the position of its end effector. Homogeneous transformations are good tools to achieve this goal.

A homogeneous transformation is a matrix that combines rotation and translation to describe the motion of the manipulator.

- Rotation matrix: represents orientations
- Translation matrix: represents translations

Homogenous transformations are a combination of both.
Representing positions

Consider figure 4 where we have a rotation in the plane $(x, y)$. Our goal is to find the relationship between the two reference frames $(O_0x_0y_0)$ and $(O_0x_1y_1)$. Consider reference frame $O_0x_0y_0$ and let

$$R^0_1 = [x^0_1 | y^0_1]$$

be the coordinates of $x_1, y_1$ in reference frame $O_0x_0y_0$. We have

$$x^0_1 = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

therefore

$$y^0_1 = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$

In this notation, the superscript denotes the reference frame. $R^0_1$ is a convention and is called rotation matrix. It is important to note that

- $R^0_1$ corresponds to rotation of reference 1 with respect to 0
- $R^0_0$ corresponds to rotation of reference 0 with respect
The relationship between $R_0^1$ and $R_0^0$ is

$$R_0^1 = (R_0^0)^{-1}$$

Other properties of the rotation matrix are:

- $R^T = R^{-1}$
- $\det R = 1$

**Example**

Consider figure 5, find the coordinates of point $p$ in the $(x_1, y_1)$ reference frame.

**Solution**

Let $p^0$ be the coordinate of point $p$ in frame $O_0x_0y_0$ and let $p^1$ be the coordinate of point $p$ in frame $O_1x_1y_1$, note that $O_0$ is the same as $O_1$ in this case.

$$p^0 = R_0^1 p^1$$

and

$$p^1 = R_0^0 p^0$$

Therefore:

$$R_0^1 = \begin{bmatrix} \cos(70) & -\sin(70) \\ \sin(70) & \cos(70) \end{bmatrix}$$

and finally

$$p^1 = \begin{bmatrix} 4.5292 \\ -3.6724 \end{bmatrix}$$

**Rotation in 3D**

Consider figure 6, the goal is to determine the rotation matrix about the $z$-axis by an angle $\theta$. The rotation matrix in this case is given by

$$R_{z,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

and

$$R_{y,\theta} = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

Any rotation matrix satisfies the following properties

- $R_{z,0} = I$
- $R_{x,\theta} R_{x,\beta} = R_{x,\theta+\beta}$
- $(R_{z,\theta})^{-1} = R_{z,-\theta}$

Any rotation matrix $R$ is 3 by 3 matrix. We write $R \in SO(3)$, where SO stands for special orthogonal. Even though a rotation matrix has nine elements, only three variables are used to specify the rotation. This implies that rotation matrices have redundancies. The standard direction of rotation is counter clockwise.
Importance of the rotation matrices
A rotation matrix can be used in three different ways:
• Provide relationship between two different frames.
• Transform the coordinates of a point from one frame to another frame
• Determine the new coordinates of the same point in the same coordinate frame after a rotation.
Manipulators: common arrangements

Fig. 7. Common robotic manipulator arrangement

Fig. 8. Workspace for some robotic manipulators