Problem 1: Exploring a maze using reinforcement learning

We want an autonomous robot to explore the maze shown in figure 1 using the Bellman equation. The instantaneous numerical values of the value function at each state are shown in figure 1. The robot is still in the training or exploration phase. The robot is in state 3e, calculate the value function at this state. We assume random and uniform transition rules. We assume the discount factor is 0.5.

Problem 2: Polynomial trajectory planning

The robot in figure 2 is moving in straight line in the x-direction. We want the robot to reach points A, B, C and D with the desired speeds shown in table 1. Design polynomial trajectory to achieve the desired motion.

Problem 3: Potential field method for differential drive robot

A robot is navigating to reach a desired position as shown in figure 3. The robot is using the potential field method where the region of influence of the obstacles is equal to 10m for all obstacles. Obtain the numerical values of the repulsive force for all obstacles.

Problem 4: Differential drive robot

A differential drive robot has wheels with different diameters. The left wheel radius is 5cm and the right wheel radius is 4cm. The distance between the wheels is 15cm. Table 2 shows the numerical values of a differential drive robot orientation angle $\theta(t)$ as well as its linear velocity components in the x- and y- directions.

![Table 2: Time evolution of the robot's orientation angle and linear velocity components](image)

Fig. 1. Maze and numbers of the value function associated with each cell
Table 1: Speed and position constraints

<table>
<thead>
<tr>
<th>Point</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>0m</td>
<td>5m</td>
<td>8m</td>
<td>14m</td>
</tr>
<tr>
<td>Speed</td>
<td>0m/s</td>
<td>Not important</td>
<td>Not important</td>
<td>3m/s</td>
</tr>
<tr>
<td>Time</td>
<td>0</td>
<td>Not important</td>
<td>Not important</td>
<td>6s</td>
</tr>
</tbody>
</table>

Fig. 2. Robot moving in straight line

Fig. 3. Robot workspace with obstacles

1) Sketch the differential drive robot. Make sure the robot looks nice.
2) Calculate the angular velocity of the robot in rad/s.
3) Calculate the linear speed of the robot in m/s.
4) Find the angular velocity of each wheel in rad/s.
5) What is the relationship between the wheels’ angular velocities to make the robot move in a straight line? Recall that the wheels have different radii.

Problem 5: Circular motion of differential drive

We want to plan the path of a differential drive robot to turn around a point located at 2m from its initial position. We want the robot to complete the full circle in 31.4160s. Calculate the angular speed of each wheel in rad/s. The robot’s wheels have a radius of 5cm and the distance between them in 15cm.
Problem 6: Differential drive robot and encoders

We use encoders to obtain the angular and linear speeds of a differential drive mobile robot. The wheels' diameter is $10\text{cm}$ and the distance between the wheels is $15\text{cm}$.

1) Briefly, explain the working principle of encoders, use drawings.
2) Assuming we use an incremental encoder with four sections, what is the resolution of the encoder?
3) The output of the right and left wheel encoders is shown in figure 5. Are we using an incremental or an absolute encoder? explain.
4) What is the angular velocity of each wheel in $\text{rad/s}$?
5) Calculate the angular and linear speed of the robot.
6) Assuming that the robot starts at the origin $(x, y) = (0, 0)$ with zero orientation angle ($\theta = 0$) and maintains constant speeds equal to the speeds of the previous question, what is the robot's position and orientation after $5s$?
Fig. 5. Wheel encoder reading