Problem 1
Consider the tank system shown in figure 1-top. We treat $q_0$ as a disturbance. The output is the liquid level. The pipe has a square shape with area $1 \times 5 cm^2$. The inflow is controlled by the valve which permits or restricts the flow. The position of the valve is $p$ and it is related to the inflow rate by the following equation

$$p = N_0 q_i$$  \(1\)

The inflow rate ($q_i$) is proportional to the position of the valve ($p$). We take $N_0 = 1$ (SI unit) in this case. This simply means that when $p = 1 cm$, the inflow rate is $1 cm^3/s$. The maximum value for the valve position is 5cm. That is

$$0 < p \leq 5 cm$$ \(2\)

This means that the maximum inflow rate is $5 cm^3/s$. The valve position $p$ is the control variable. The sampling time is $T = 0.1 s$.

Part 1: Tank system simulation
Consider the block diagram of figure 2-top. Its corresponding Simulink diagram is shown in figure 3-top.
1) Find the transfer function of the ZOH and the analog subsystem.
2) Find the final value of the liquid level when $q_i = 1 cm^3/s$ and $q_0 = 0.8 cm^3/s$. Use two different methods:
   - The final value theorem or the expression of $y(k)$.
   - The Simulink block diagram of figure 3-top.

Part 2: Open loop control
1) Now consider the open loop system shown in figure 2-bottom. Show that

$$Y(z) = \frac{T}{z-1} (K N_0 R(z) - Q_0(z))$$ \(3\)

2) Assuming that $r = 5 cm$ and $q_0 = 0.8 cm$, find the final value for the liquid level, you can solve by hand or use Simulink.

Part 3: Closed loop and proportional controller design
1) Now, we consider the closed loop system shown in figure 1-bottom. The corresponding Simulink block diagram is shown in figure 3-bottom. This block diagram has a saturation block. Explain the role of the saturation block and the numerical values of the upper and lower limits.
2) Show that

$$Y(z) = \frac{T}{z-1} (K N_0 R(z) - Y(z) - Q_0(z))$$ \(4\)

3) Calculate the interval of the gain for the stability of the closed loop system.
4) Write the steady state error as a function of the gain, $R(z)$ and $Q_0(z)$.
5) How to reduce the steady state error? (use the results from the previous question)
6) Calculate the gain so that the steady state error is less than 0.8 cm when $r = 5 cm$ and $q_0 = 0.8 cm^3/s$.
Fig. 3. Simulink Block diagrams. Top: The tank system, middle: open loop system, and bottom: closed loop system for $K = 2.5$ and $K = 5$.

7) Build the simulink block diagram and simulate the closed loop system for $K = 5$. Simulate the system with and without saturation. Discuss your results and compare.