EEE 188: Homework Assignment #7

Date: 03/30/2017
Due: 04/06/2017

Problem 1

Consider the tank system shown in figure 1-top. We treat $q_0$ as a disturbance. The output is the liquid level. The pipe has a square shape with area $1 \times 5 \text{cm}^2$. The inflow is controlled by the valve which permits or restricts the flow. The position of the valve is $p$ and it is related to the inflow rate by the following equation

$$p = N_0 q_i$$

The inflow rate ($q_i$) is proportional to the position of the valve ($p$). We take $N_0 = 1$ (SI unit) in this case. This simply means that when $p = 1 \text{cm}$, the inflow rate is $1 \text{cm}^3/s$. The maximum value for the valve position is $5 \text{cm}$. That is

$$0 < p \leq 5 \text{cm}$$

This means that the maximum inflow rate is $5 \text{cm}^3/s$. The valve position $p$ is the control variable. The sampling time is $T = 0.1 \text{s}$.

Part 1: Tank system simulation

Consider the block diagram of figure 2-top. Its corresponding Simulink diagram is shown in figure 3-top.

1) Find the transfer function of the ZOH and the analog subsystem.
2) Find the final value of the liquid level when $q_i = 1 \text{cm}^3/s$ and $q_0 = 0.8 \text{cm}^3/s$. Use two different methods:
   - The final value theorem or the expression of $y(k)$.
   - The Simulink block diagram of figure 3-top.

Part 2: Open loop control

1) Now consider the open loop system shown in figure 2-bottom. Show that

$$Y(z) = \frac{T}{z-1} \left( K N_0 R(z) - Q_0(z) \right)$$

2) Assuming that $r = 5 \text{cm}$ and $q_0 = 0.8 \text{cm}$, find the final value for the liquid level, you can solve by hand or use Simulink.

Part 3: Closed loop and proportional controller design

1) Now, we consider the closed loop system shown shown in 1-bottom. The corresponding Simulink block diagram is shown in figure 3-bottom. This block diagram has a saturation block. Explain the role of the saturation block and the numerical values of the upper and lower limits.
2) Show that

$$Y(z) = \frac{T}{z-1} \left( K N_0 R(z) - Y(z) - Q_0(z) \right)$$

3) Calculate the interval of the gain for the stability of the closed loop system.
4) Write the steady state error as a function of the gain, $R(z)$ and $Q_0(z)$.
5) How to reduce the steady state error? (use the results from the previous question)
6) Calculate the gain so that the steady state error is less than $0.8 \text{cm}$ when $r = 5 \text{cm}$ and $q_0 = 0.8 \text{cm}^3/s$
Fig. 3. Simulink Block diagrams. Top: The tank system, middle: open loop system, and bottom: closed loop system for $K = 2.5$ and $K = 5$

7) Build the simulink block diagram and simulate the closed loop system for $K = 5$. Simulate the system with and without saturation. Discuss your results and compare.