Problem 1: INFANTE AUV

Autonomous underwater vehicles are robotic submarines that can be used for a variety of studies of the underwater environment. A picture of the AUV of interest called INFANTE AUV is shown in figure 1. The vertical and horizontal dynamics of the vehicle must be controlled to remotely operate the AUV. The variables of interest in the horizontal motion are sway speed and the yaw angle. A linear model of the horizontal plane motion of the vehicle is given by

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} -0.14 & -0.69 & 0 \\ -0.19 & -0.048 & 0 \\ -0.14 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0.056 \\ -0.23 \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x
\end{align*}
\]

where \(x_1\) is the sway speed, \(x_2\) is the yaw angle, and \(x_3\) is the yaw rate. \(u\) is the rudder deflection. Obtain a discrete-state space model for the system with a sampling period of \(50\,ms\).

1) By hand, obtain a discrete-state space model with a sampling period of \(50\,ms\).
2) Use Matlab function c2d for the same sampling period to obtain the discrete time system. Compare with the previous question.
3) Plot the time evolution of the sway speed and the yaw angle when the input is a unit step. Pick appropriate initial conditions.


Problem 2: River pollution (textbook page 286)

To monitor river pollution, we need to model the concentration of biodegradable matter contained in the water in terms of biochemical oxygen demand for its degradation. We also need to model the dissolved oxygen deficit defined as the
difference between the highest concentration and the actual concentration in mg/l. If the two variables of interest are the state variables $x_1$ and $x_2$, respectively, then an appropriate model is given by

$$\dot{x} = \begin{bmatrix} -k_1 & 0 \\ k_1 & -k_2 \end{bmatrix} x$$ (3)

where $k_1$ is a biodegradation constant and $k_2$ is a reaeration constant, and both are positive. Assume that the two positive constants are unequal.

1) Obtain a discrete time model for the system with a sampling period $T$.
2) Simulate the discrete system for the normalized values of the parameters $k_1 = 1, k_2 = 2$, with sampling periods of $T = 0.01s$ and $T = 0.1s$. Pick appropriate initial conditions. You may use command initial to solve.

**Hint**

The parameters of function lsim are the system (in this case discrete state space), the input, and the time. For example:

```matlab
T_s = 0:Ts:Tf; u = ones(1,Tf/Ts+1); lsim(sys,u,T)
```

- $T_s$ is the sampling period and $T_f$ is the final time. The total number of samples is $T_f/T_s + 1$.
- Make sure you keep the same sampling period.
- In this case the input is a unit step, $T_f/T_s + 1$ is the number of samples.